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HORIZONTAL GRADIENT EFFECTS ON  
DIRECTIONAL ACOUSTICAL SENSORS

JAMES WARD PIGMAN

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HORIZONTAL GRADIENT EFFECTS ON  
DIRECTIONAL ACOUSTICAL SENSORS

by

James Ward Pigman  
Lieutenant, United States Naval Reserve  
B.S., Birmingham-Southern College, 1958



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# ABSTRACT

An investigation was made into the types of fixing error that would result when two passive directional sensors in one velocity medium was obtaining its fix on a target in another velocity medium. For typical velocity differences that could exist in the oceans off the U. S., this error was investigated for various values of sensor spacing and target/sensor relationships to a velocity discontinuity. Tabular and graphical results were obtained which revealed that errors could result under certain conditions and these results were predictable and could operationally be avoided.



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## 1. Introduction.

Though fixing by means of cross bearings from two or more directional acoustical sensors is an established concept in Naval Warfare, there has been little investigation done as to the effects of fixing error due to horizontal water gradients. With the advent of the SSQ-36 sonobuoy a tool is now available to the ASW type aircraft that allows measurement of the temperature of the water.

Sound waves travel in straight lines only in a medium in which the speed is everywhere constant. The speed of sound in sea water depends on salinity, pressure, and temperature. The problem of salinity and pressure can be neglected when discussing horizontal gradients in open ocean environments since both will be almost constant throughout the media. With decreasing temperature, the water becomes more dense, and the velocity of the sea water decreases. A useful equation for relating these variables is

$$V_{ft}/sec = 4502 + 9.184T - 0.0289T^2 + 0.0182Y \quad (1-1)$$

where Y = depth in positive feet

and T = temperature.

Using (1-1) gives a velocity of about 4900 ft/sec or 1500 m/sec for temperatures of 50°F. Similarly, a temperature of 70°F and 90°F gives about 1525 m/sec (5000 ft/sec) and 1550 m/sec (5100 ft/sec) respectively.

An operational problem was envisioned in which a submarine was trying to penetrate a barrier that had been established either in or outside of the Gulf Stream. The submarine was then outside of, or in the Gulf Stream. This would imply a very distinguishable temperature gradient existed between the buoys and the submarine



which would imply that a distinct horizontal velocity gradient existed.

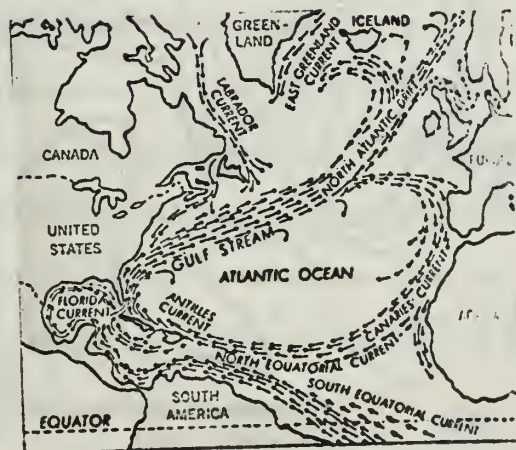
In the ocean are various currents. Many of these currents are of a different temperature than the water they are flowing through. At this point a digression will be made to give a short explanation of currents to the reader. Although it is not absolutely necessary for understanding the problem of horizontal gradients and its solution, it will allow a better understanding of the mechanisms involved.

The two primary causes of currents are the exchange of heat with the atmosphere and energy supplied by winds. Even in the absence of winds, a thermal circulation would be set up because heating in lower latitudes would warm the surface water, causing them to expand slightly, whereas cooling in high latitudes would cause them to shrink. The waters in low latitudes would therefore stand a little higher and there would be a flow away from the equator towards higher latitudes. It is believed that the winds have a much more important effect. Not only do they exert a drag on the surface of the sea, setting the surface layers in motion but they also will bring about a redistribution of mass establishing slopes. [2]

However, the intense currents such as the Gulf Stream have been shown not to be related to local winds. It results from the variation of Coriolis Force with latitude. The winds only add energy. As an example of the size of a major current, the Gulf Stream starts out in Florida about 95 nautical miles wide and nearly 2 miles deep with a speed of 3.5 knots. Off Cape Hatteras it has been reduced to 1 knot, and becomes wider, more shallow and more interrupted as it moves out into the Atlantic. The average temperature of the surface water is 80°F as far north as Cape Hatteras and the eastern side is warmer than the western. Figure 1 shows the Gulf Stream and its different component currents. Figure 2a shows the intricate



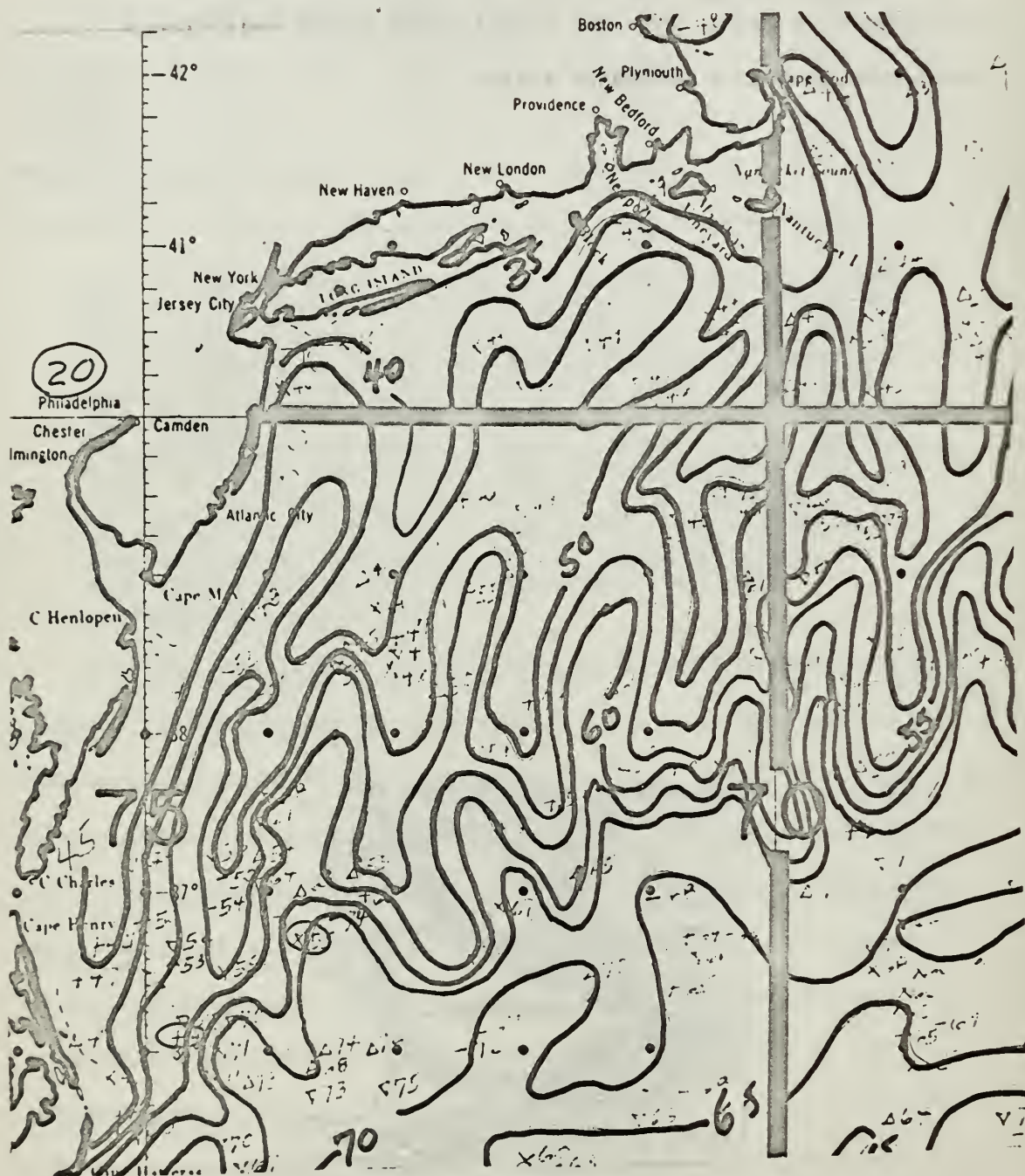
isotemps of sea surface temperatures for a four day period. To simplify the problem a first order approximation is made as illustrated in Figure 2b. The temperatures of the Gulf Stream are broken down into blocks or strips.



Formation and course of the Gulf Stream. [2]

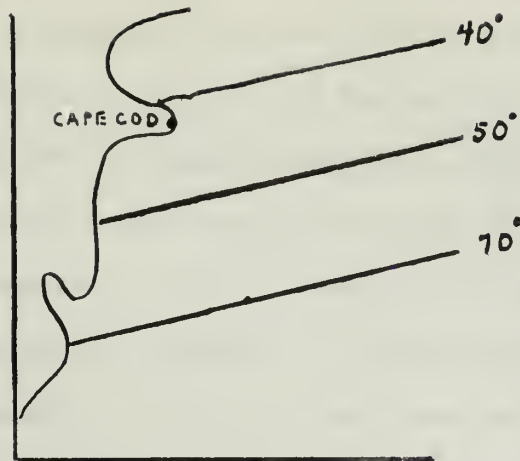
Formation and Course of the Gulf Stream

Figure 1



SST 8-12 January 1964

Figure 2a



Approximations of Sea Surface Temperatures

Figure 2b

## 2. Problem Modeling.

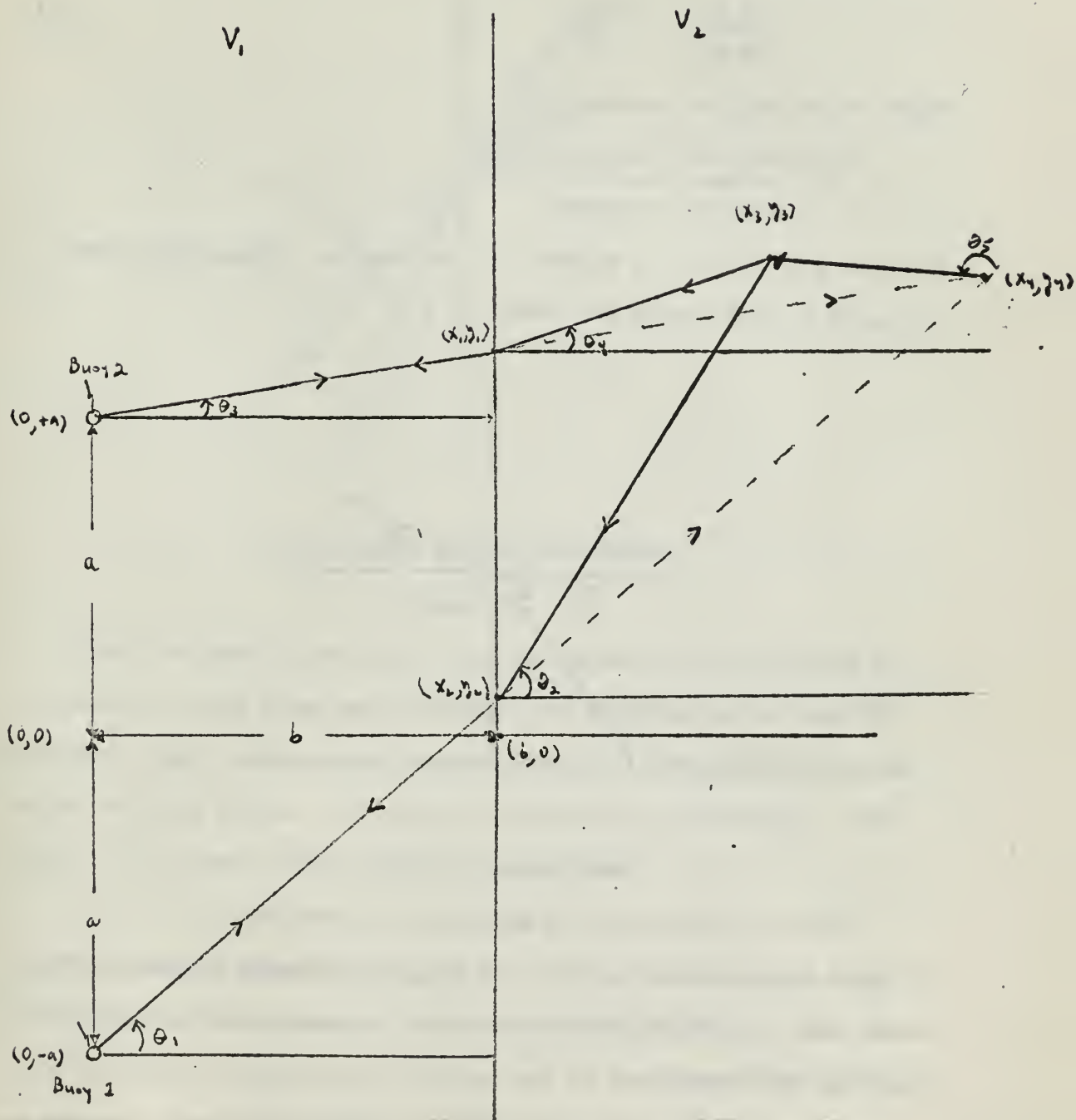
When structuring the problem the following assumptions were made:

- (1) An initial detection was already given; thus a detection range limitation was not considered. It made no difference if detection was the result of bottom bounce, channeling, direct beam, or surface reflection. This allows vertical gradients to be neglected since they have almost no effect on the problem when a detection is given. The difference between horizontal gradients at various depths does not change appreciably.
- (2) A directional acoustic sensor was considered to have no bearing error, i.e., finite beamwidth. Adding this problem would be the next order of sophistication.

(3) Although both horizontal continuous and discontinuous gradients occur in the ocean, the temperature gradients were considered sharp and distinct, i.e., discontinuous.

Figure 3 exhibits a pictorial display of the problem. Two sonobuoys are placed  $2a$  apart from each other and perpendicular to a distinct temperature gradient which is a distance  $b$  away. The origin  $(0,0)$  of the problem is the midpoint between the two sonobuoys. Consider the sound source actually at point  $(X_3, Y_3)$ . For buoy 1 the sound ray path starts with an angle  $\theta_2$  to the normal and when it reaches the gradient, the ray bends toward the normal and becomes  $\theta_1$ . Similarly for buoy 2 the ray path starts with an angle  $\theta_4$  to the normal and then is bent to become  $\theta_3$ .





Horizontal Ray Bending Due to Different Velocity Media

Figure 3

When a sound source must pass through a velocity differential the acoustical Snell's Law can be applied to understand what happens. Snell's Law says

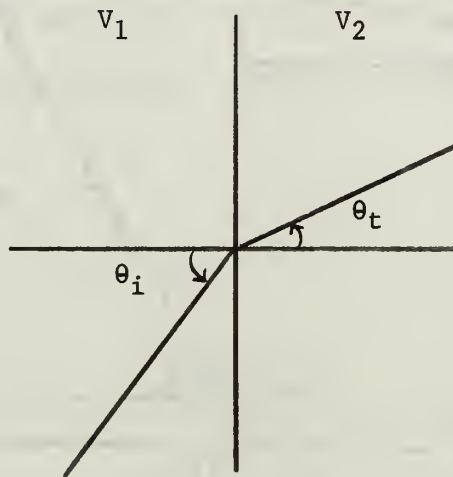
$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{V_1}{V_2} \quad (2-1)$$

where  $\theta_i$  = angle of incident ray

$\theta_t$  = angle of transmitted ray

$V$  = velocity of medium.

If  $V_1$  is greater than  $V_2$  Figure 4 is applicable. This implies that the angle is bent toward the normal or  $\theta_t < \theta_i$ .



Snell's Law ( $V_1 > V_2$ )

Figure 4

If  $V_2$  is greater than  $V_1$ , (i.e.,  $V_1 < V_2$ ) then Figure 5 is applicable and we have the unusual phenomenon which is described as a critical angle  $\theta_c$  occurring when

$$\sin \theta_c = \frac{V_1}{V_2} \quad (2-2)$$

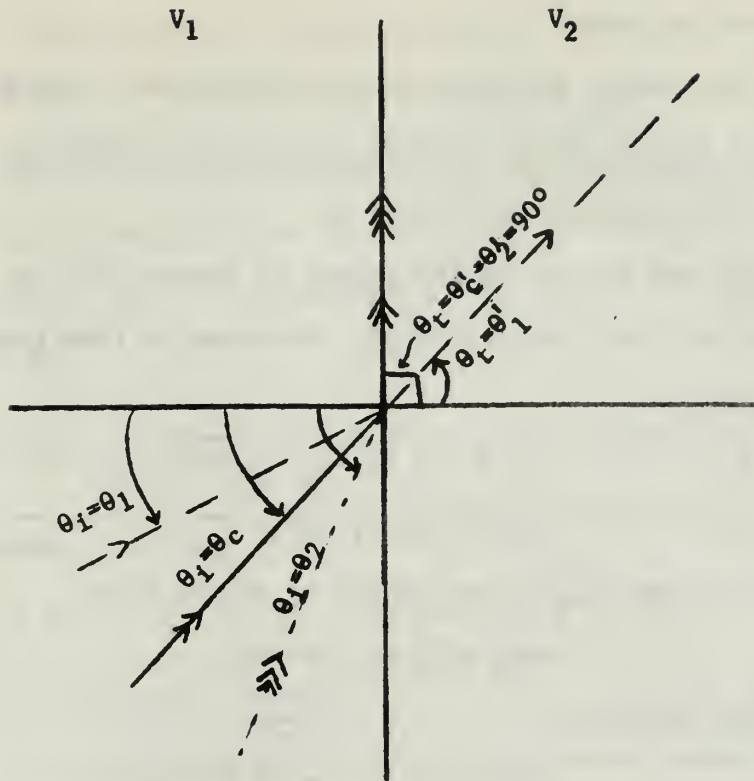


Illustration of Critical Angles

Figure 5

Here the angle is bent away from the normal or  $\theta_t < \theta_i$  until  $\theta_c$  is reached at that point and thereafter the incident ray is bent  $90^\circ$  (critical angle) and no sound (theoretically) is transmitted into the second velocity medium. In reality some sound is transmitted. However, it is so small that it will be neglected.

If it is assumed that the reader is at the sensor, the rays enter with angles  $\theta_1$  and  $\theta_3$  (Figure 3). Projecting their path backward the sound source appears to be at the point  $(X_4, Y_4)$ . Here the error involved is arbitrarily selected and is the vector from  $(X_4, Y_4)$  to  $(X_3, Y_3)$ . The error angle  $\theta_5$  is taken from the apparent extension of the ray with angle  $\theta_3$  and the error vector. Finally if the case arises where  $X_3 \leq b$ , then  $(X_3, Y_3) = (X_4, Y_4)$  since  $V_1 = V_2$  and no error exists.

### 3. Mathematical Solution.

Now that the actual and apparent sound paths have been shown, it is necessary to derive the accompanying equations in order to solve the problem. For reference see Figure 3.

Let  $(X_2, Y_2)$  and  $(X_1, Y_1)$  be the points of intersection at  $\underline{b}$  of ray path 1 and ray path 2 respectively. Equation of line from  $(0, -a)$  to  $(X_2, Y_2)$  is

$$Y = X \tan \theta_1 - a \quad (3-1)$$

or since  $X_2 = b$ ,  $Y_2 = b \tan \theta_1 - a \quad (3-1a)$

Equation of line from  $(X_2, Y_2)$  to  $(X_3, Y_3)$  is of the form

$$Y - Y_2 = (X - X_2) \tan \theta_2 \quad (3-2)$$

Combining (3-1a) and (3-2)

$$Y - b \tan \theta_1 + a = (X - b) \tan \theta_2 \quad (3-3)$$

Previously it was shown that Snell's Law says,

$$\theta_2 = \arcsin \left( \frac{V_2}{V_1} \sin \theta_1 \right) \quad (3-4)$$

Combining (3-3) and (3-4) gives the equation of line  $\underline{A}$

$$Y - b \tan \theta_1 + a = (X - b) \tan \left( \arcsin \left( \frac{V_2}{V_1} \sin \theta_1 \right) \right). \quad (3-5)$$

Similarly the equation of line from  $(0, +a)$  to  $(X_2, Y_2)$  is of the form

$$Y = X \tan \theta_3 + a \quad (3-6)$$

or since  $X_2 = b$ ,

$$Y_2 = b \tan \theta_3 + a \quad (3-6a)$$

and the equation from  $(X_1, Y_1)$  to  $(X_3, Y_3)$  is

$$Y - b \tan \theta_3 - a = (X - b) \tan \theta_4 \quad (3-7)$$



Combining (3-6a) and (3-7) gives the equation of line A'

$$Y - b \tan \theta_3 - a = (X - b) \tan \left( \arcsin \left( \frac{V_2}{V_1} \sin \theta_3 \right) \right) \quad (3-8)$$

The apparent intersection  $(X_4, Y_4)$  for  $\theta_3 > 0$  is the intersection of (3-1) and (3-6) or

$$X_4 \tan \theta_1 - a = X_4 \tan \theta_3 + a \quad (3-9)$$

Solving for  $X_4$ ,

$$X_4 = \frac{2a}{\tan \theta_1 - \tan \theta_3}, \quad \theta_3 > 0 \quad (3-10)$$

Again combining (3-1) and (3-6)

$$Y_4 \tan \theta_3 + a \tan \theta_3 = Y_4 \tan \theta_1 - a \tan \theta_1 \quad (3-11)$$

$$Y_4 = \frac{a(\tan \theta_3 + \tan \theta_1)}{\tan \theta_1 - \tan \theta_3}, \quad \theta_3 > 0 \quad (3-12)$$

The real position  $(X_3, Y_3)$  for  $\theta_3 > 0$  is given by the intersection of A and A'. Rewriting equation (3-5) and (3-8),

$$Y = b \tan \theta_1 - a + X \tan \left( \arcsin \left( \frac{V_2}{V_1} \sin \theta_1 \right) \right) - b \tan \left( \arcsin \left( \frac{V_2}{V_1} \sin \theta_1 \right) \right) \quad (3-13)$$

$$Y = b \tan \theta_3 + a + X \tan \left( \arcsin \left( \frac{V_2}{V_1} \sin \theta_3 \right) \right) - b \tan \left( \arcsin \left( \frac{V_2}{V_1} \sin \theta_3 \right) \right) \quad (3-14)$$

To facilitate writing equations, the following definitions are added,

$$Z_1 = \arcsin \left( \frac{V_2}{V_1} \sin (\theta_1) \right)$$

$$Z_2 = \arcsin \left( \frac{V_2}{V_1} \sin (\theta_3) \right)$$

$$T_1 = \tan(Z_1)$$

$$T_2 = \tan(Z_2)$$

Combining (3-13) and (3-14) and solving

$$X_3 (T_1 - T_2) = b (\tan \theta_3 - \tan \theta_1) + 2a + b (T_1 - T_2)$$

$$X_3 = \frac{b(\tan \theta_3 - \tan \theta_1 + T_1 - T_2) + 2a}{T_1 - T_2}, \quad \theta_3 > 0. \quad (3-15)$$

Again combining (3-13) and (3-14) and solving

$$X_3 = \frac{Y_3 + a - b \tan \theta_1 + b T_1}{T_1} = \frac{Y_3 - a - b \tan \theta_3 + b T_2}{T_2} \quad (3-16)$$

$$Y_3 = \frac{b(T_2 \tan \theta_1 - T_1 \tan \theta_3) - a(T_1 + T_2)}{T_2 - T_1}, \quad \theta_3 > 0. \quad (3-17)$$

For the case  $\theta_3 < 0$ ,  $\underline{A}$  is the same, i.e.,

$$Y_2 = b \tan \theta_1 - a \quad (3-1a)$$

$$Y - b \tan \theta_1 + a = (X - b) T_1 \quad (3-18)$$

However  $\underline{A}'$  becomes

$$Y_2 = -b \tan \theta_3 + a \quad (3-19)$$

Combining (3-19) and (3-7) gives the equation of  $\underline{A}''$

$$Y + b \tan \theta_3 - a = (b - X) T_2 \quad (3-20)$$

Using (3-1) and (3-19) gives the apparent intersection  $(X_4, Y_4)$

for  $\theta_3 < 0$ .

$$X_4 \tan \theta_1 - a = -X \tan \theta_3 + a \quad (3-21)$$

$$X_4 = \frac{2a}{\tan \theta_1 + \tan \theta_3}, \quad \theta_3 < 0 \quad (3-22)$$

$$\frac{Y_4 + a}{\tan \theta_1} = \frac{a - Y_4}{\tan \theta_3} \quad (3-23)$$

$$Y_4 = \frac{a(\tan \theta_1 - \tan \theta_3)}{\tan \theta_1 + \tan \theta_3}, \quad \theta_3 < 0, \quad (3-24)$$

Solving now for the real intersection  $(X_3, Y_3)$  using (3-18) and (3-20),

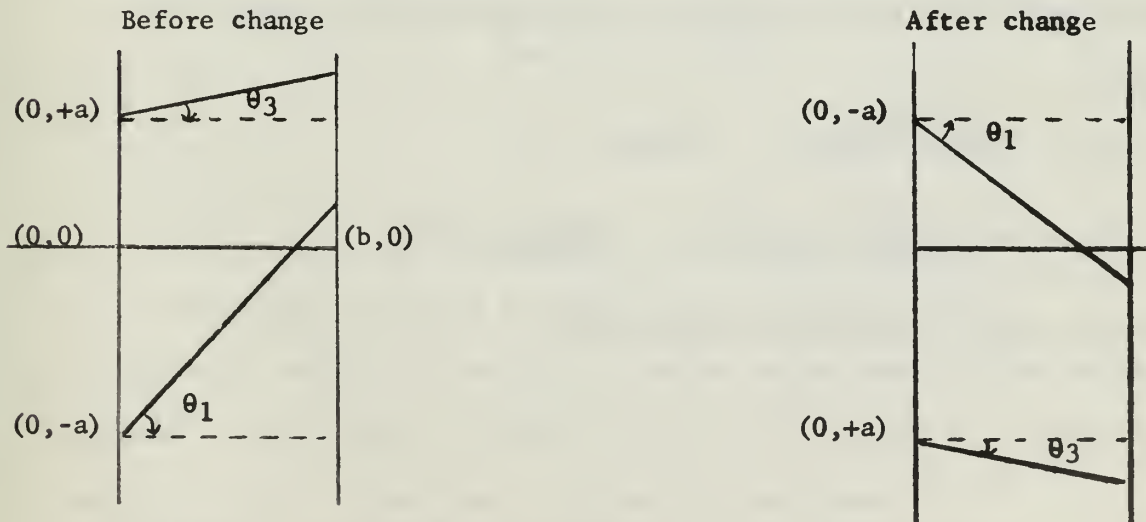
$$Y_3 = b \tan \theta_1 - a + X_3 T_1 - b T_1 = -b \tan \theta_3 + a + b T_2 - X_3 T_2 \quad (3-25)$$

$$X_3 = \frac{b(T_2 - T_1 - \tan \theta_3 - \tan \theta_1) + 2a}{T_1 + T_2}, \quad \theta_3 < 0 \quad (3-26)$$

$$X_3 = \frac{Y_3 + a + b(T_1 - \tan \theta_1)}{T_1} = \frac{a + b(T_2 - \tan \theta_3) - Y_3}{T_2} \quad (3-27)$$

$$Y_3 = \frac{a(T_1 - T_2) + b(T_2 \tan \theta_1 - T_1 \tan \theta_3)}{T_1 + T_2}, \quad \theta_3 < 0. \quad (3-28)$$

These equations are for the first quadrant. If the sound source is in the fourth quadrant make the following changes to Figure 3 as shown below in Figure 6. Also change  $\theta_2$  and  $\theta_4$  to correspond.



Changes to Problem if Sound Source is in Quadrant Four

Figure 6

Now that all the necessary equations have been developed, it is now possible to find the relative percent error as a function of  $a$ ,  $b$ ,  $\theta_1$ , and  $\theta_3$ . This is done by taking the ratio of radial error or error vector to the actual distance of the sound source from the origin (0,0). Relative error (RE) is

$$RE = \frac{\sqrt{(X_3 - X_4)^2 + (Y_3 - Y_4)^2}}{\sqrt{X_3^2 + Y_3^2}} \quad (3-29)$$

Appendix I contains the computer program from which solutions for relative error were obtained.

#### 4. Example.

To demonstrate the forementioned, the following example is worked. If two sonobuoys are placed on the inner edge of the Gulf Stream and the submarine is outside the Stream so that  $V_1 = 1525$  m/sec and  $V_2 = 1500$  m/sec,  $a = 1$  NM =  $b$  and  $\theta_1 = 30^\circ$ ,  $\theta_3 = 15^\circ$ . Using equations (3-10) and (3-12) it is possible to solve for  $X_4$  and  $Y_4$  respectively, where

$$X_4 = \frac{(2) (1)}{.57735 - .26795} = \frac{2}{.3094} = 6.46$$

$$Y_4 = \frac{(1) (.57735 + .26795)}{.57735 - .26795} = \frac{.8453}{.3094} = 2.73$$

and the apparent position is (6.46, 2.73).

Using equation (3-15) and (3-16) it is possible to solve for  $X_3$  and  $Y_3$  respectively, where

$$Z_1 = .51416$$

$$Z_2 = .25741$$

$$T_1 = .56483$$

$$T_2 = .26325$$

$$X_3 = \frac{(1) (.26795 - .57735 + .56483 - .26325) + (2) (1)}{.56483 - .26325} = \frac{2 - .00782}{.30158}$$

$$= 6.61$$

$$Y_3 = \frac{(1) (.26325) (.57735) - (.56483) (.26795) - (1) (.56483 + .26325)}{-.30158}$$

$$= \frac{(.15199 - .15135) - .82808}{-.30158} = 2.74$$

and the actual position is (6.61, 2.74).

To figure relative error, the relationship

$$RE = \frac{\sqrt{(6.61 - 6.46)^2 + (2.74 - 2.73)^2}}{\sqrt{(6.61)^2 + (2.74)^2}} = 1.99\%$$

is used.

## 5. Results.

Basic results are graphically illustrated in Figures 7 through 12 and tabulated in tables 1 through 5 for varying  $\theta_1$  and  $\theta_3$ , different  $V_1$  and  $V_2$  and varying A/B ratios which show relative error. Percent relative error is plotted versus  $\theta_3$ , for given  $\theta_1$ , velocities and a/b ratios. In general it can be observed that:



(a)  $a/b$  is invariant; that is, only the ratio of  $a/b$  effects the results, not the value of  $a$  or  $b$  alone. For example, if  $a$  is 1 mile and  $b$  is 2 miles or if  $a$  is 50,000 yards and  $b$  is 100,000 yards the relative error is the same.

(b) it was noted that there was not a large difference in error whether  $V_1 = 1500$  and  $V_2 = 1525$  or if  $V_1 = 1525$  and  $V_2 = 1500$ . However, there was a consistently smaller relative error for  $V_1 > V_2$  compared to  $V_2 > V_1$ .

(c) for a ratio of  $a/b = 1$ , it was noticed that the error was independent of  $\theta_1$ . This is not true for  $a/b$  either less than or greater than 1. For  $\theta_3 \leq 0^\circ$  the relative error was about 1.5% which gradually increased to 3% for  $\theta_3 = 30^\circ$ . Over  $30^\circ$  error increased fairly rapidly up to about 10% for  $\theta_3 = 55^\circ$ . At around  $70^\circ$  the angle became critical for  $V_1 = 1525$  and  $V_2 = 1500$ .

(d) for  $a/b > 1$ , for example  $a/b = 2$ , sonobuoys spaced 20 miles apart and the buoys placed 5 miles from the gradient, the error increased as  $\theta_1$  increased holding  $\theta_3$  constant. The error also increased as  $\theta_3$  increased. As the ratio of  $a/b$  increased the error increased. It should be noted that around  $\theta_3 = 25^\circ$  the error started increasing fairly rapidly. The error at its extreme reached over 12%. Average error appeared to be around 3%.

(e) for  $a/b < 1$ , the reverse of  $a/b > 1$  was observed. As  $\theta_1$  increased, the error became smaller except at the maximum extremes for  $\theta_1$  and  $\theta_3$ . That is, unless  $\theta_1 - \theta_3 < 10^\circ$  as  $\theta_1$  increases, error is smaller. A major reason for this is that as  $a/b \rightarrow 0$ , most of the apparent positions fall in  $V_1$ , so that error is 0.

Relative Error in Miles for A/B Ratio of  
1,  $V_1 = 1500\text{m/s}$ ,  $V_2 = 1525\text{m/s}$ .

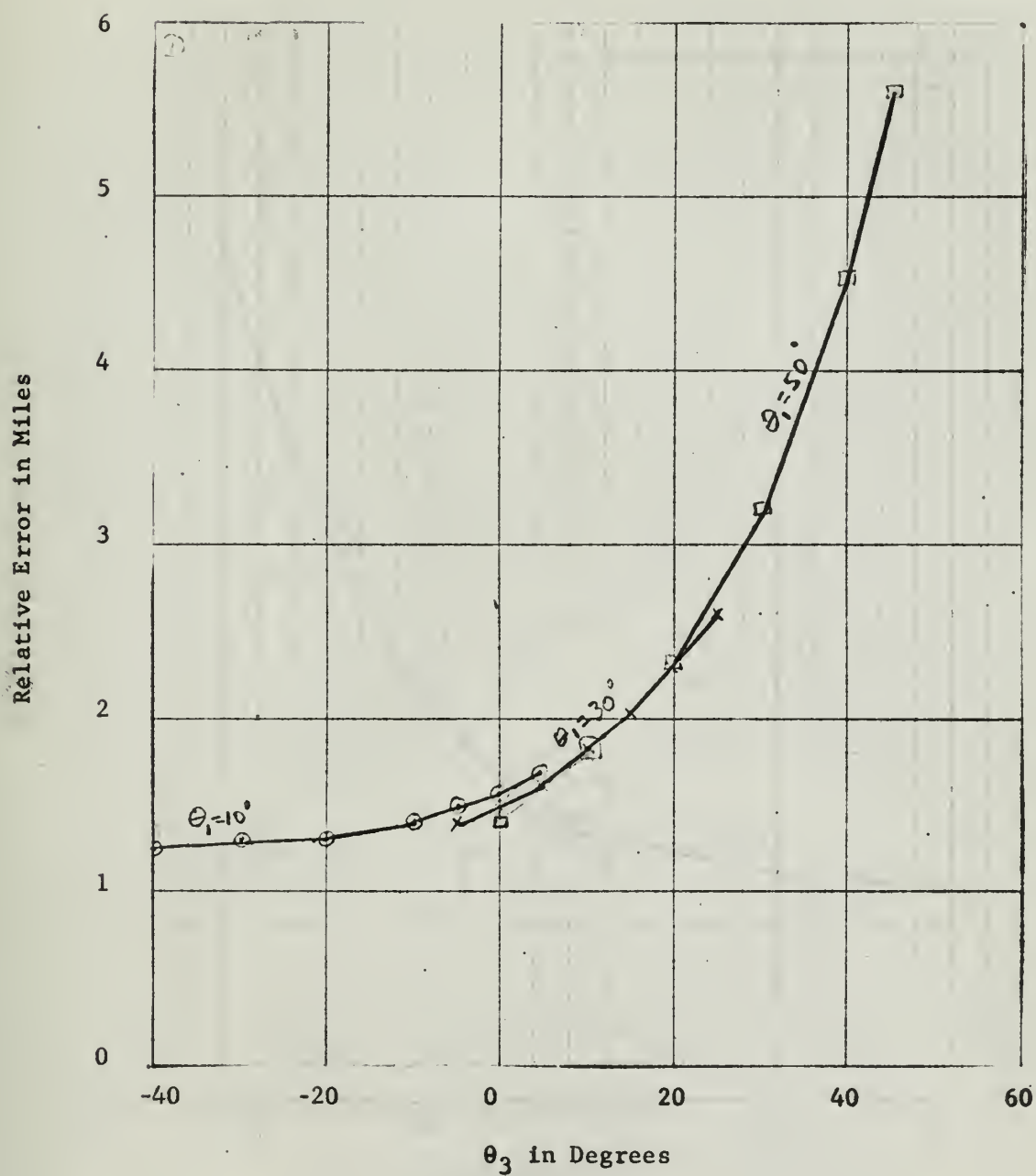


Figure 7

Relative Error in Miles for A/B Ratio of  
1,  $V_1 = 1525\text{m/s}$ ,  $V_2 = 1500\text{m/s}$ .

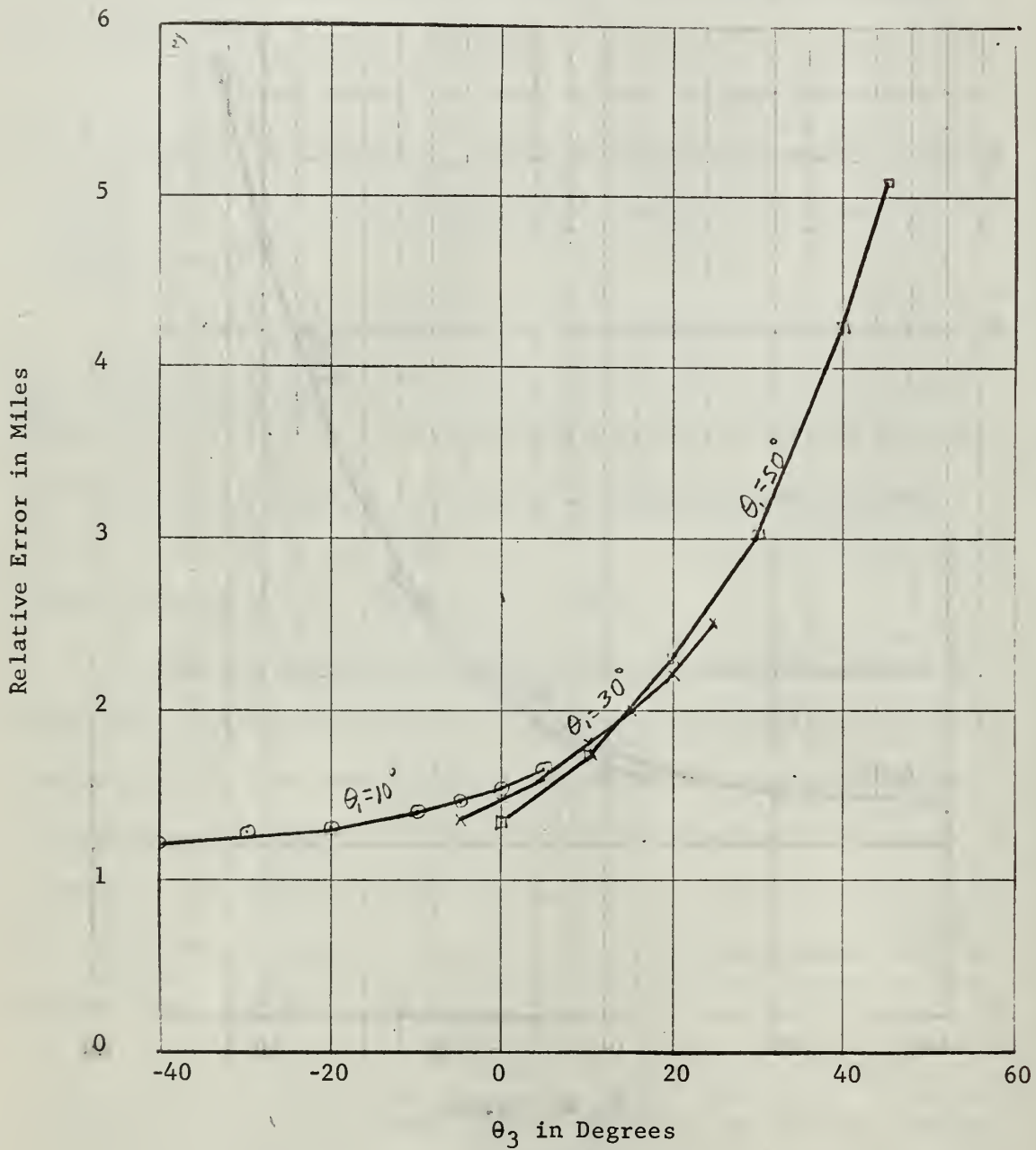


Figure 8



Relative Error in Miles for A/B Ratio of  
 $5/1$ ,  $V_1 = 1500\text{m/s}$ ,  $V_2 = 1525\text{m/s}$ .

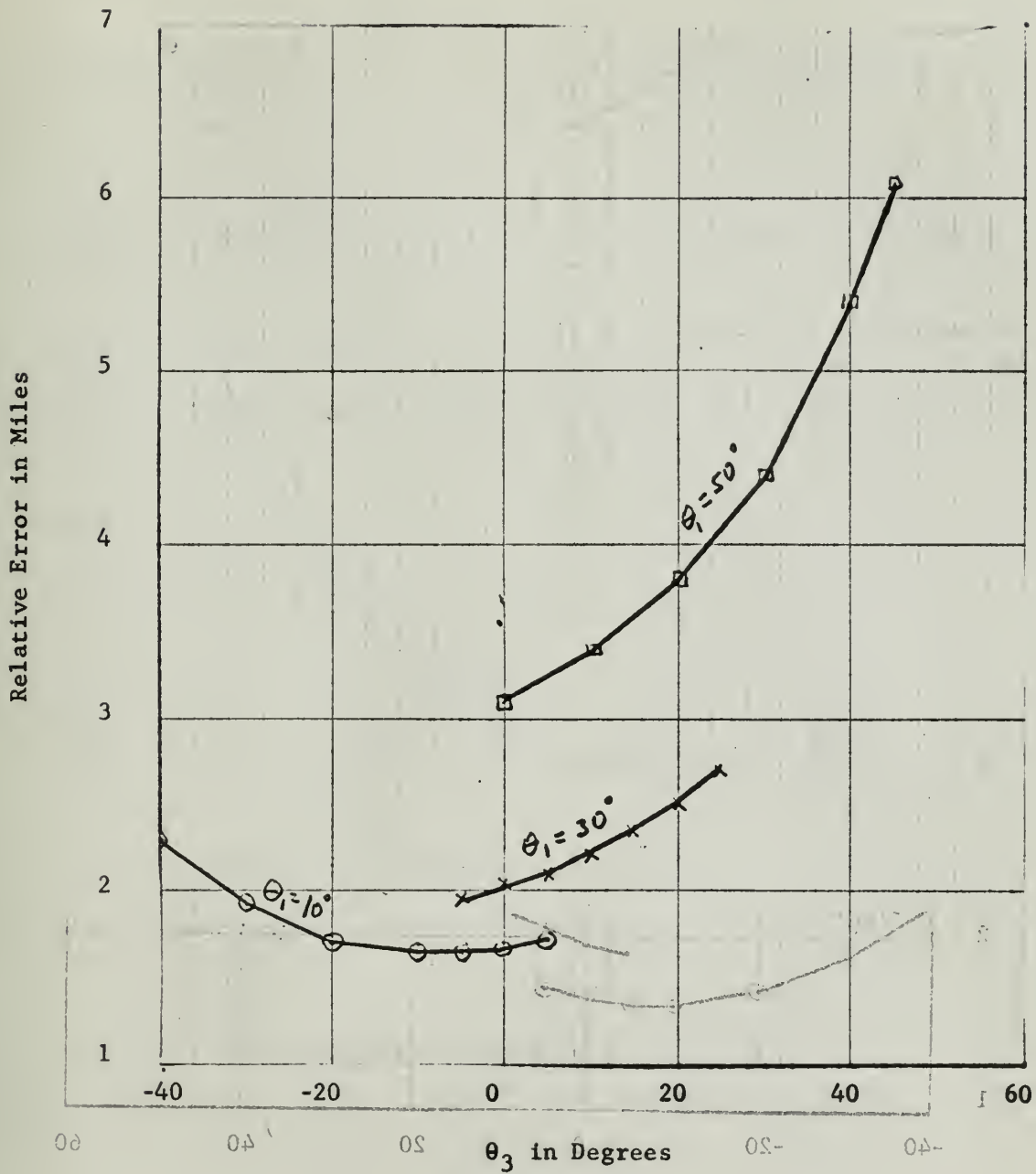


Figure 9

Figure 10

Relative Error in Miles for A/B Ratio of  
 $5/1$ ,  $V_1 = 1525\text{m/s}$ ,  $V_2 = 1500\text{m/s}$ .

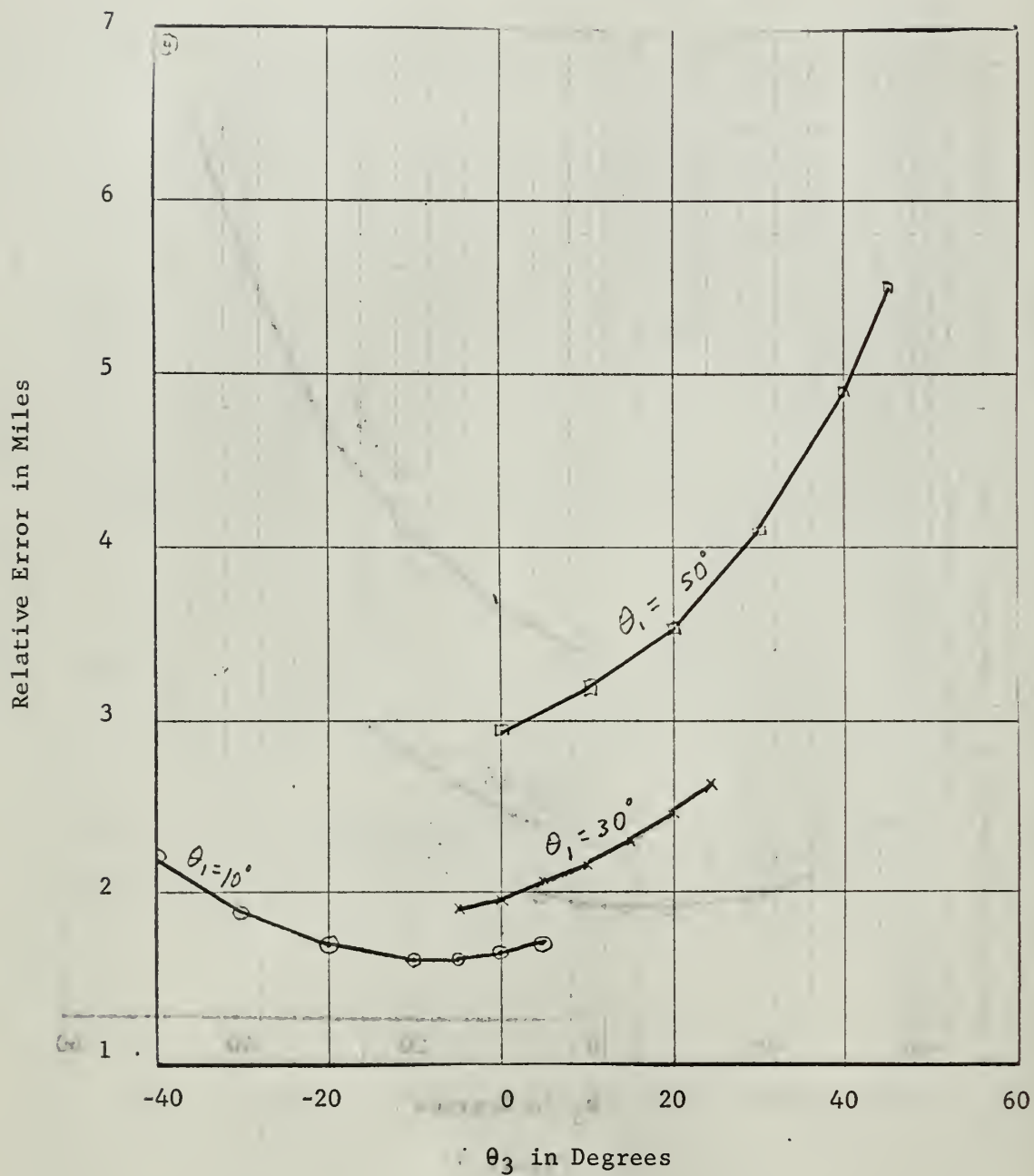


Figure 10

Relative Error in Miles for A/B Ratio of  
 $1/5$ ,  $V_1 = 1500\text{m/s}$ ,  $V_2 = 1525\text{m/s}$ .

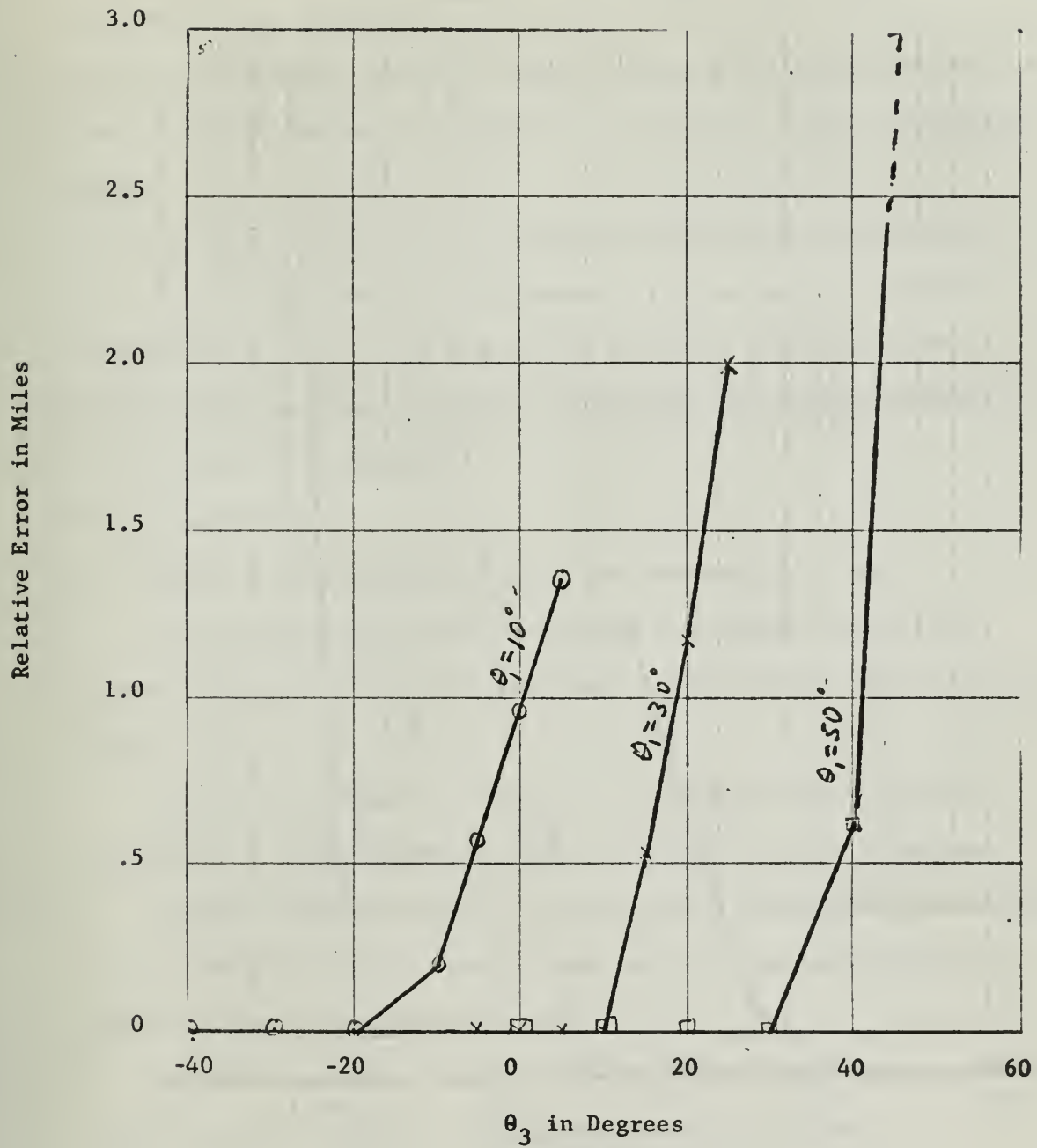


Figure 11

Relative Error in Miles for A/B Ratio of  
 $1/5$ ,  $V_1 = 1525\text{m/s}$ ,  $V_2 = 1550\text{m/s}$ .

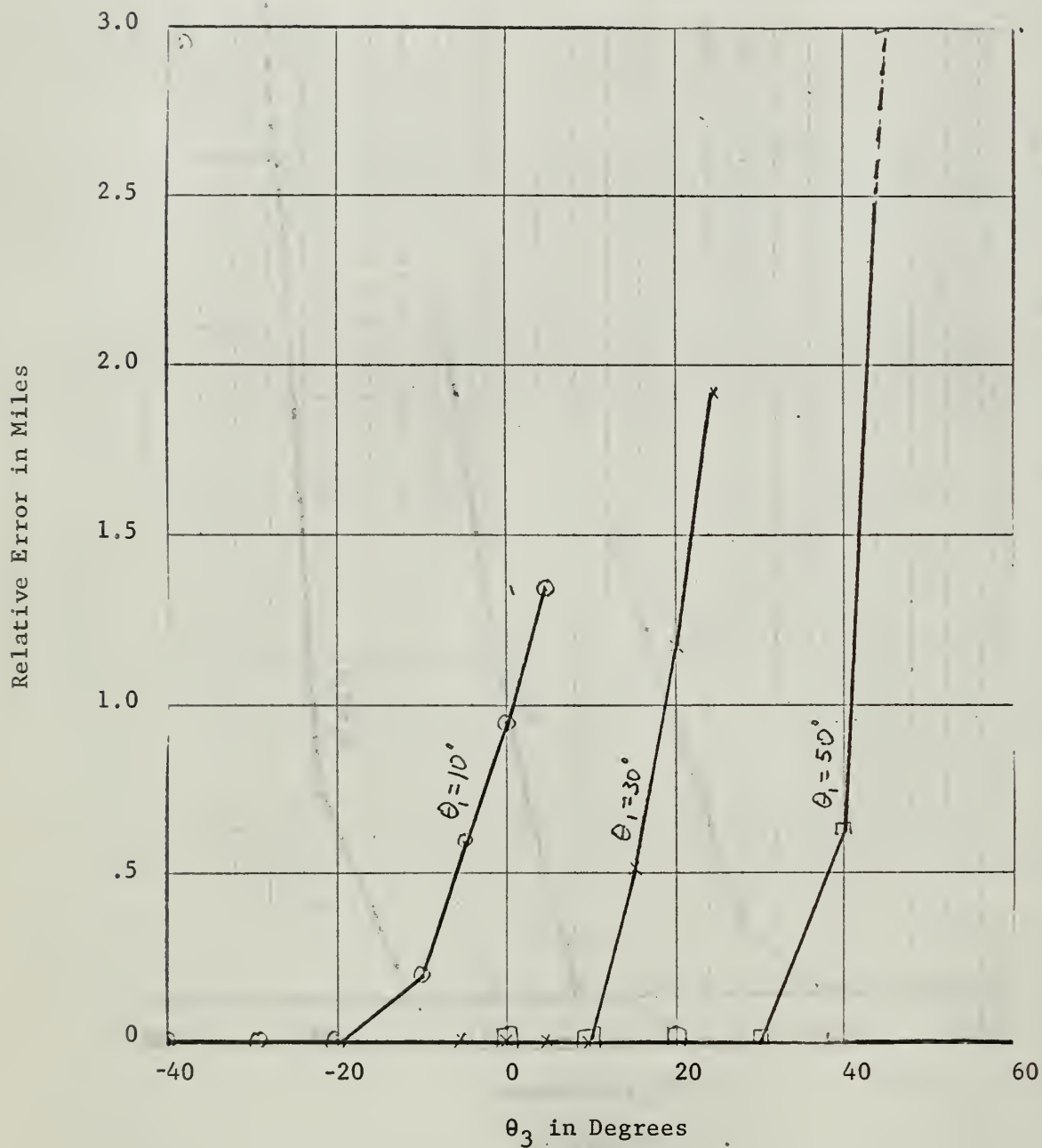


Figure 12

Although relative error allowed an effective way to represent graphically the fundamental relationship, it is also important to look at the value of absolute error. This error is tabulated in Table 6 and in Figures 13 through 18. In general it can be observed that:

(a) for  $a/b \leq 1$  there are no errors of more than 1 NM.

(b) for  $a/b > 1$  the absolute error is around .75 NM or less for  $\theta_3/\theta_1 \leq 1/2$ . If  $\theta_3/\theta_1 > 1/2$  the error goes up to 5 NM. Figure 19 is inserted to illustrate what happens to relative error if the velocity is doubled.

## 6. Conclusions.

Based on theoretical results it can be concluded that:

(a) Errors of 5% or more never take place where angles less than 40 degrees normal to the temperature discontinuity ( $13^\circ$  difference) occur.

(b) Sonobuoy spacing can affect the realm of errors possible. In general, a small sonobuoy spacing will keep error to a minimum.

(c) The farther sonobuoys are kept from a temperature discontinuity, the smaller the relative fixing errors will be, though the absolute error will not necessarily decrease.

(d) If a reasonable model of the water's velocity structure is known, the true position of the target could be obtained.

Absolute Error in Miles for A/B Ratio of  
1,  $V_1 = 1500\text{m/s}$ ,  $V_2 = 1525\text{m/s}$ .

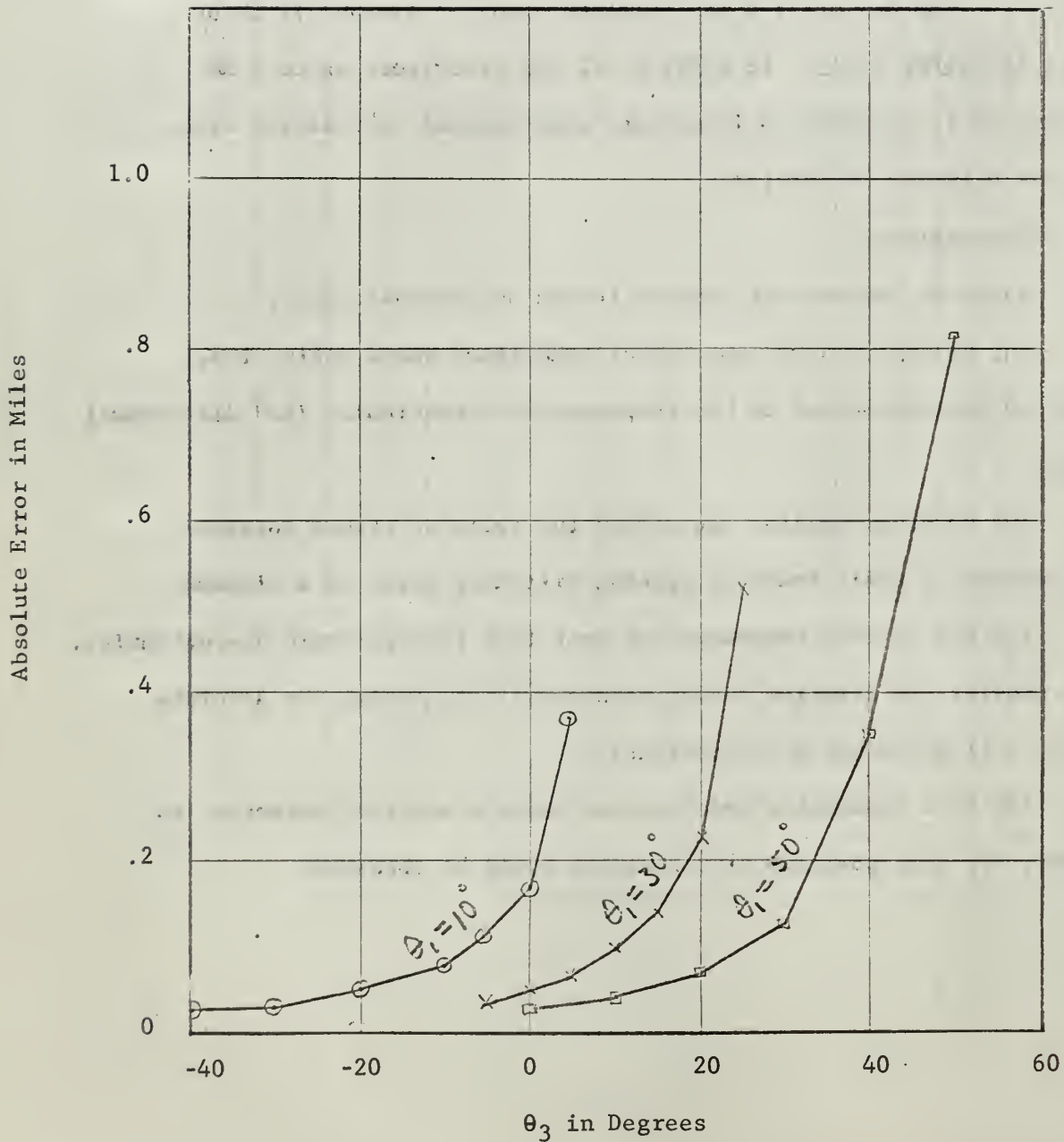


Figure 13



Absolute Error in Miles for A/B Ratio of  
1,  $V_1 = 1525\text{m/s}$ ,  $V_2 = 1500\text{m/s}$ .

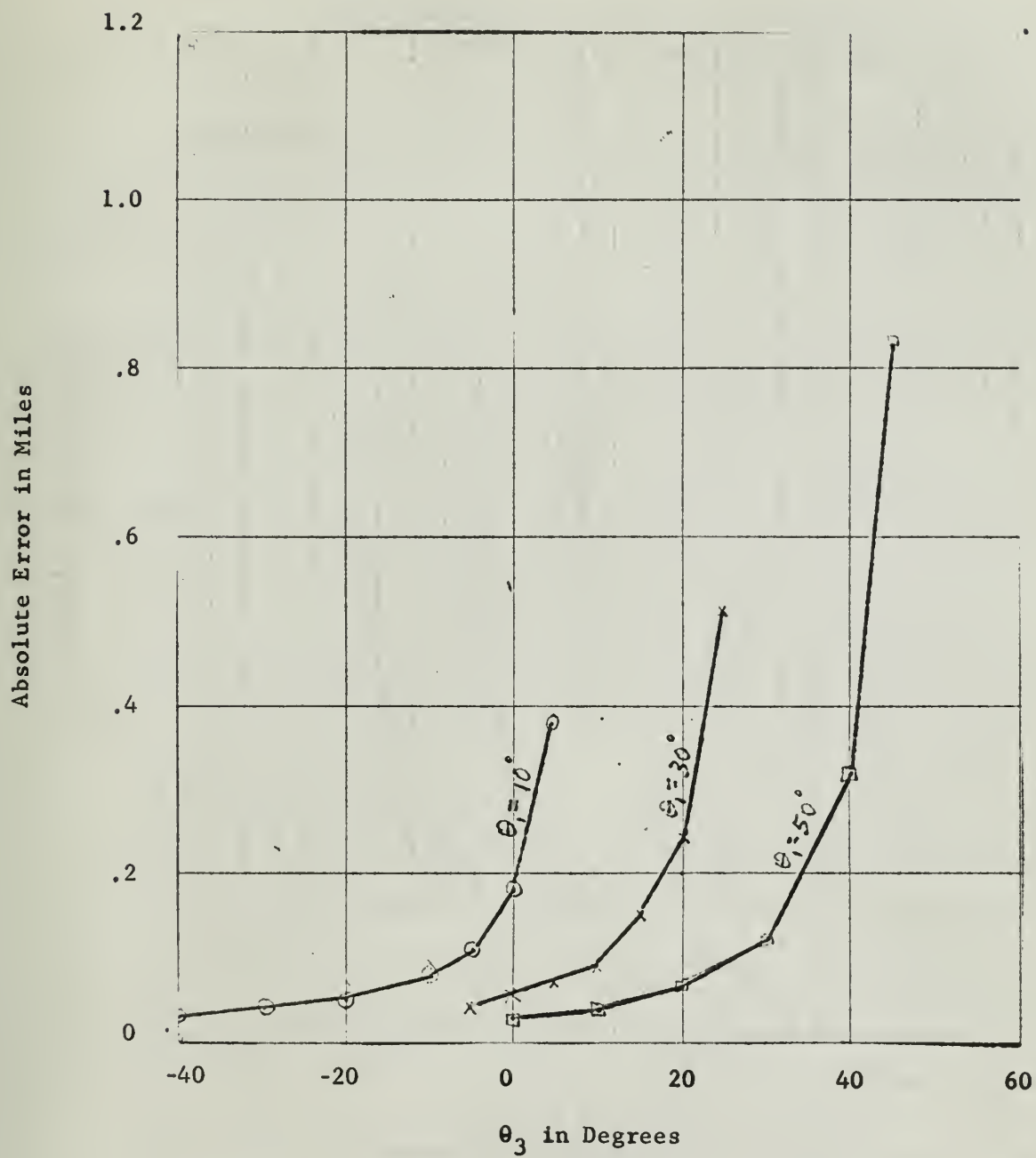


Figure 14

Absolute Error in Miles for A/B Ratio of  
 5/1,  $V_1 = 1500\text{m/s}$ ,  $V_2 = 1525\text{m/s}$ .

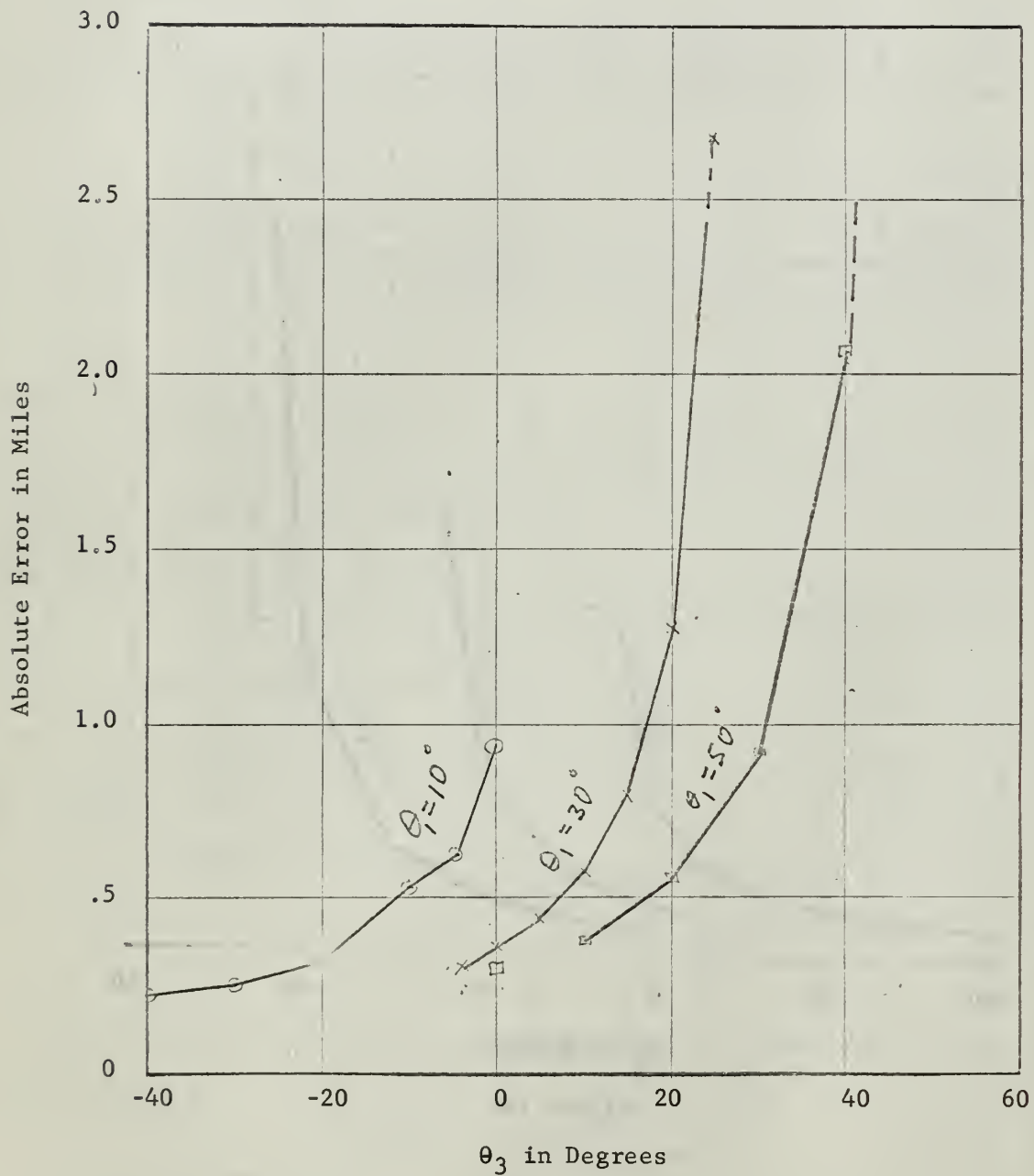


Figure 15



Absolute Error in Miles for A/B Ratio of  
 $5/1$ ,  $V_1 = 1525\text{m/s}$ ,  $V_2 = 1500\text{m/s}$ .

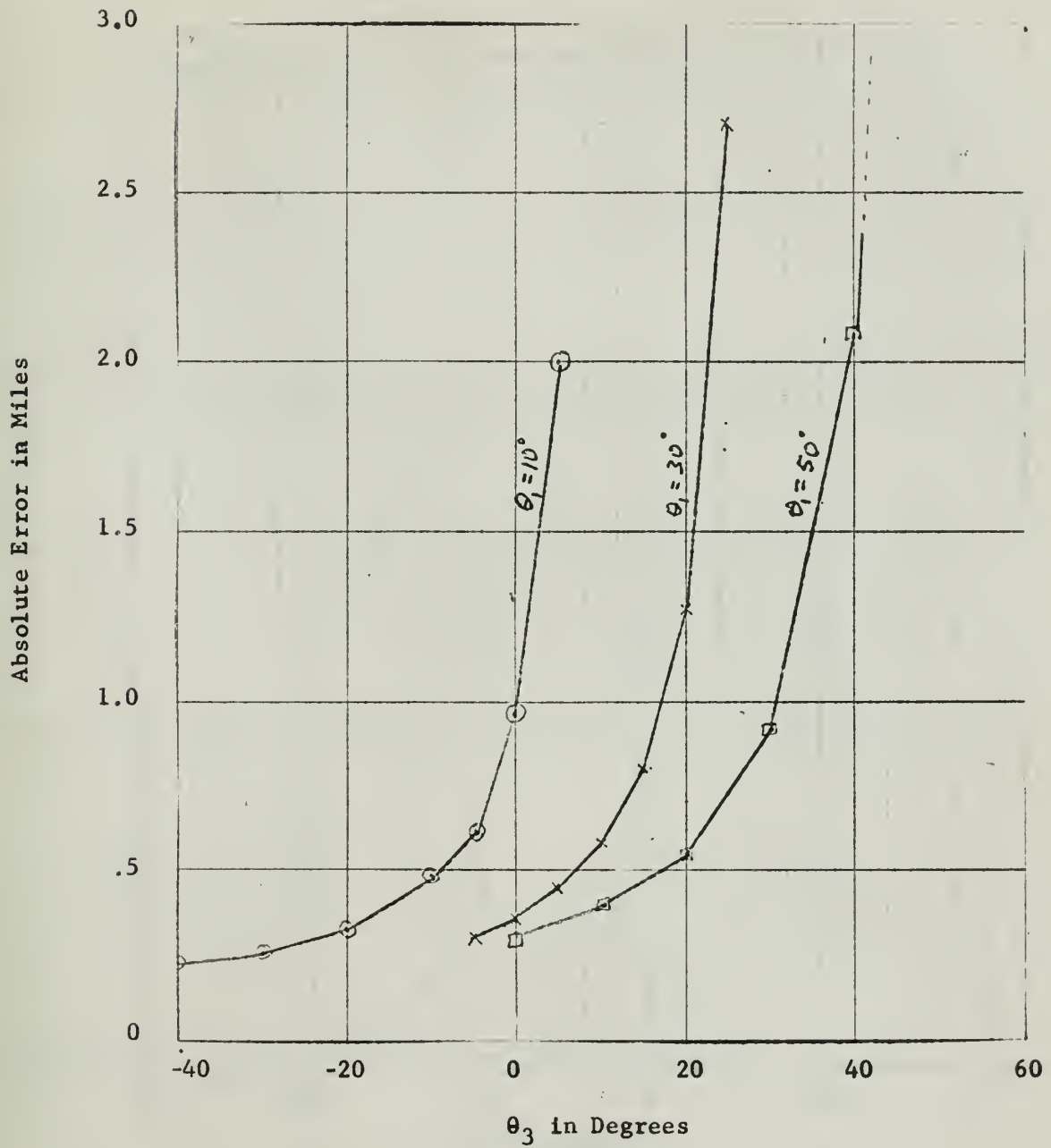


Figure 16

Absolute Error in Miles for A/B Ratio of  
1/5,  $V_1 = 1500\text{m/s}$ ,  $V_2 = 1525\text{m/s}$ .

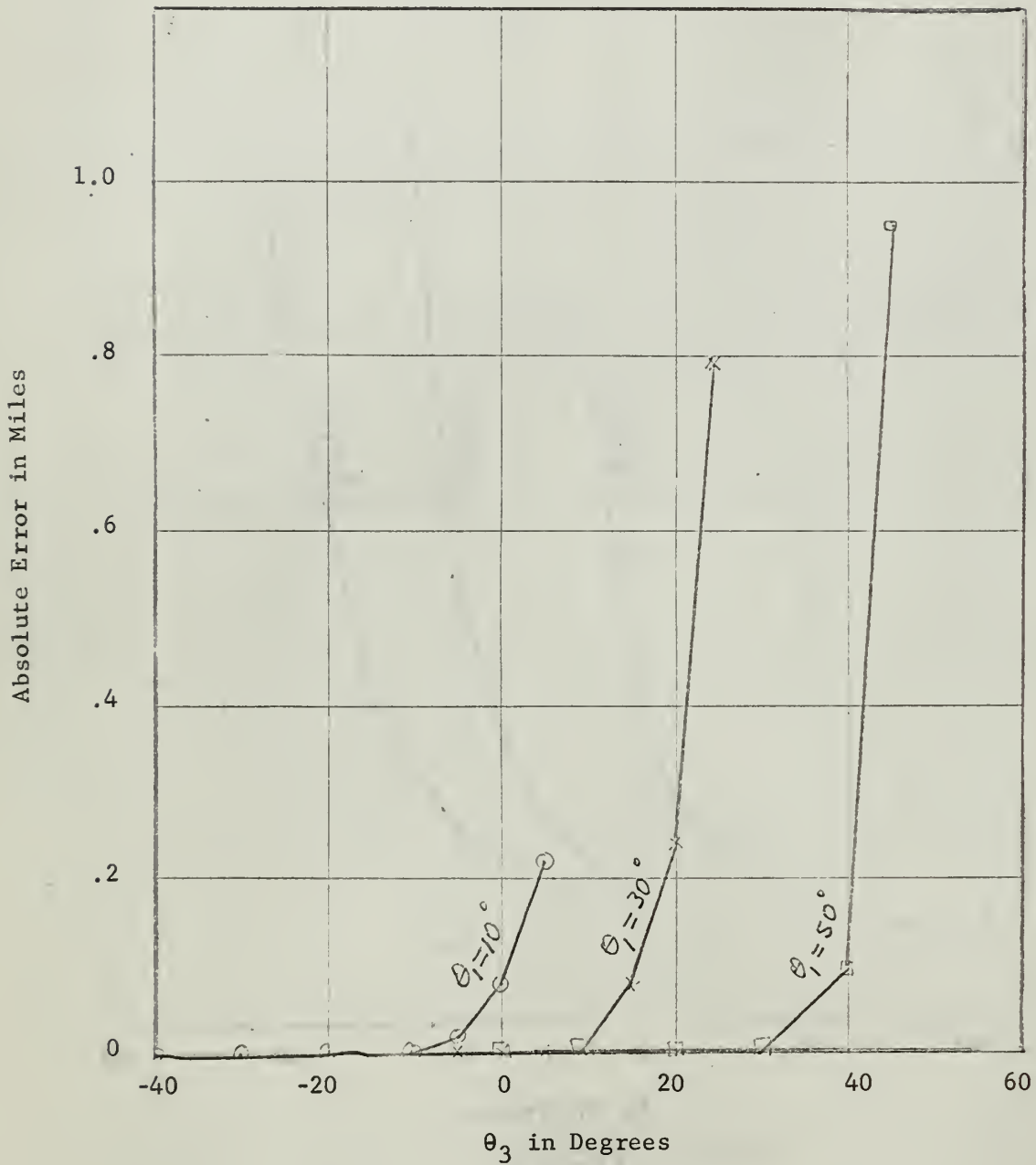


Figure 17

Absolute Error in Miles for A/B Ratio of  
 $1/5$ ,  $V_1 = 1525\text{m/s}$ ,  $V_2 = 1500\text{m/s}$ .

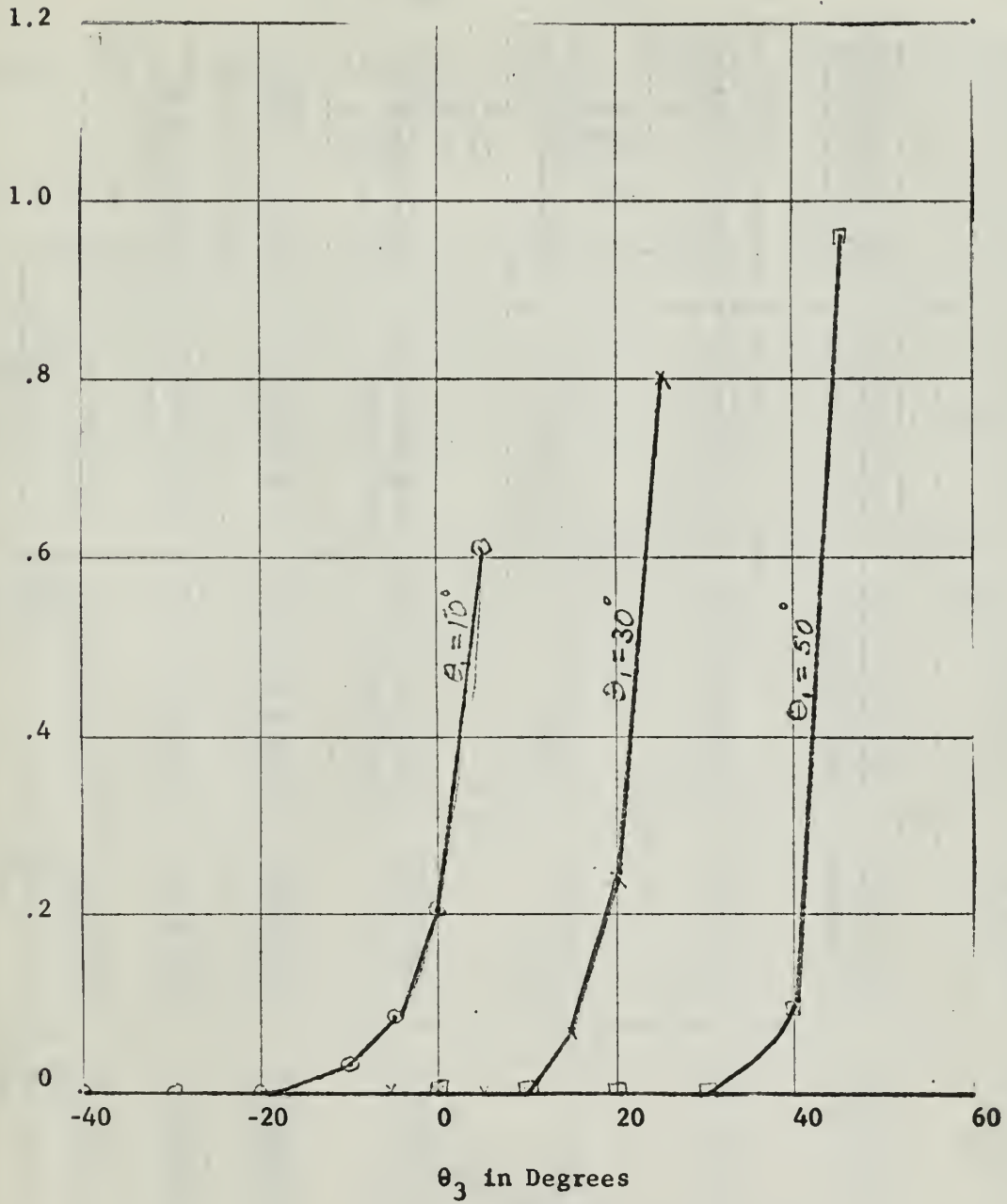


Figure 18

Relative Error in Miles for A/B of  
1,  $V_1 = 1500\text{m/s}$ ,  $V_2 = 1550\text{m/s}$ .

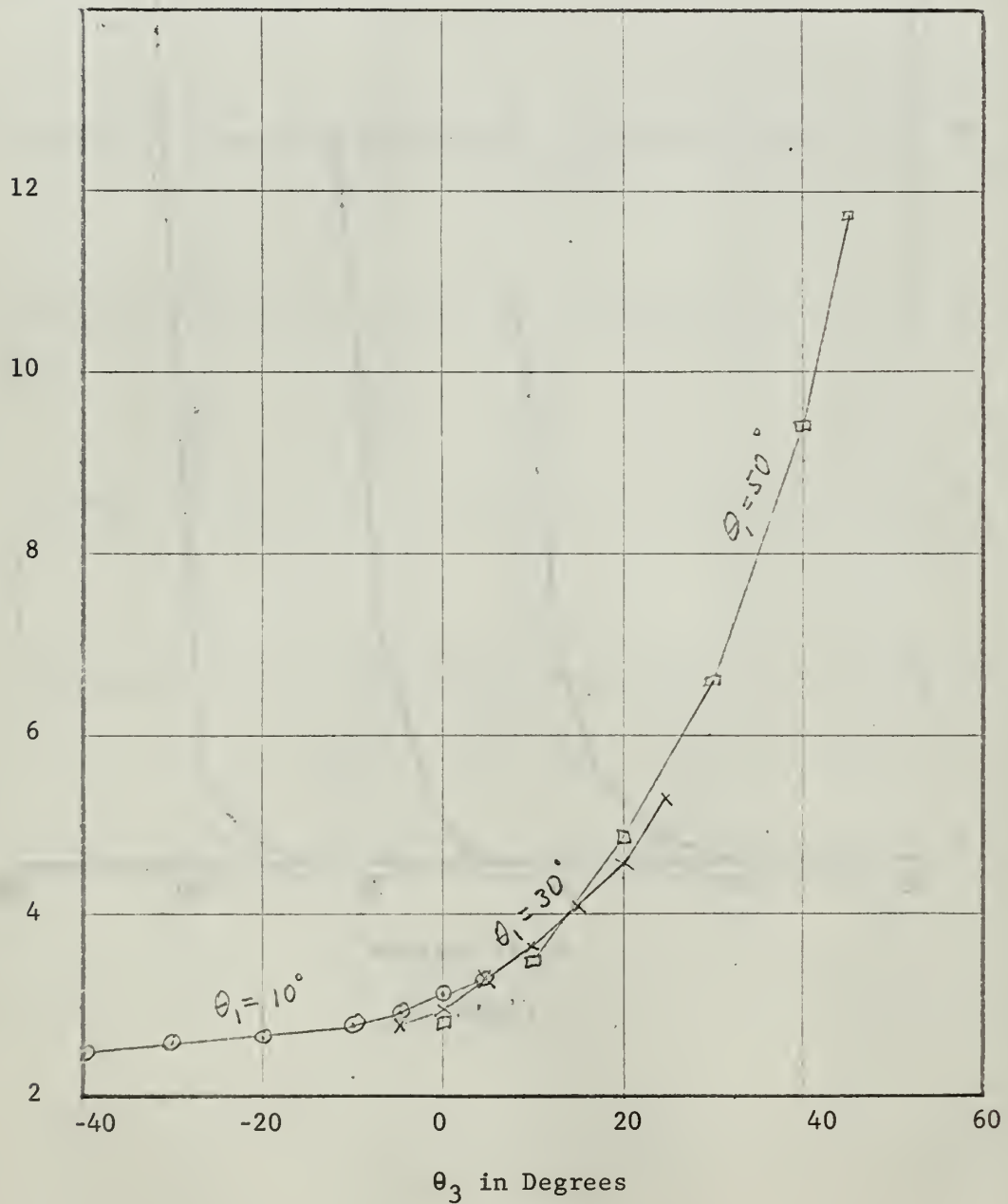


Figure 19

TABLE 1  
RELATIVE ERROR  
 $V_1 = 1500$        $V_2 = 1525$

$\theta_1 = 10^\circ$	$\theta_3$	A/B=1	A/B=1/2	A/B=1/5	A/B=1/10	A/B=2/1	A/B=5/1
1.	-40°	1.24%	0%	0%	0%	1.89%	2.28%
2.	-30	1.28	.50	0	0	1.68	1.92
3.	-20	1.33	.83	0	0	1.58	1.73
4.	-10	1.41	1.11	.20	0	1.57	1.66
5.	- 5	1.48	1.25	.57	0	1.59	1.66
6.	0	1.56	1.41	.95	.20	1.64	1.68
7.	5	1.66	1.59	1.35	.96	1.70	1.73

---

$\theta_1 = 20^\circ$	8.	-10	1.33	.83	0	0	1.58	1.73
	9.	- 5	1.41	1.00	0	0	1.62	1.75
	10.	0	1.52	1.18	.16	0	1.69	1.79
	11.	5	1.64	1.38	.58	0	1.78	1.86
	12.	10	1.79	1.61	1.04	.12	1.89	1.95
	13.	15	1.98	1.88	1.57	1.07	2.03	2.06

---

$\theta_1 = 30^\circ$	14.	- 5	1.39	.70	0	0	1.75	1.96
	15.	0	1.52	.90	0	0	1.83	2.02
	16.	5	1.67	1.12	0	0	1.94	2.11
	17.	10	1.84	1.37	0	0	2.07	2.21
	18.	15	2.04	1.66	.54	0	2.23	2.35
	19.	20	2.29	2.01	1.18	0	2.43	2.52
	20.	25	2.60	2.44	1.98	1.21	2.68	2.72

---

$\theta_1 = 40^\circ$	21.	0	1.53	.42	0	0	2.09	2.43
	22.	10	1.89	.94	0	0	2.37	2.66
	23.	20	2.39	1.64	0	0	2.77	3.00
	24.	30	3.14	2.66	1.23	0	3.39	3.53
	25.	35	3.68	3.39	2.55	1.18	3.82	3.90

---

$\theta_1 = 50^\circ$	26.	0	1.41	0	0	0	2.47	3.12
	27.	10	1.83	0	0	0	2.80	3.39
	28.	20	2.39	.69	0	0	3.26	3.79
	29.	30	3.22	1.79	0	0	3.96	4.41
	30.	40	4.57	3.56	.63	0	5.08	5.39
	31.	45	5.61	4.99	3.16	.24	5.92	6.11

---

$\theta_1 = 55^\circ$	32.	0	1.19	0	0	0	2.71	3.64
	33.	10	1.65	0	0	0	3.06	3.93
	34.	20	2.25	0	0	0	3.56	4.37
	35.	30	3.12	.79	0	0	4.32	5.05
	36.	40	4.53	2.60	0	0	5.53	6.14
	37.	50	7.17	6.15	3.21	0	7.69	8.00

$$V_1 = 1500$$

$$V_2 = 1525$$

$\theta_1=60^\circ$	$\theta_3$	$A/B=1$	$A/B=1/2$	$A/B=1/5$	$A/B=1/10$	$A/B=2/1$	$A/B=5/1$
	38. 0°	.69%	0%	0%	0%	2.95%	4.34%
	39. 10	1.19	0	0	0	3.33	4.65
	40. 20	1.83	0	0	0	3.88	5.14
	41. 30	2.74	0	0	0	4.70	5.90
	42. 40	4.22	.79	0	0	6.02	7.13
	43. 50	6.99	4.29	0	0	8.38	9.24
$\theta_1=65^\circ$	44. 0	0	0	0	0	3.09	5.30
	45. 10	.11	0	0	0	3.51	5.64
	46. 20	.79	0	0	0	4.11	6.19
	47. 30	1.73	0	0	0	5.01	7.07
	48. 40	3.24	0	0	0	6.46	8.49
	49. 50	6.07	.52	0	0	9.07	10.95
	50. 55	12.81	9.18	0	0	14.71	15.88



TABLE 2  
RELATIVE ERROR  
 $V_1 = 1525$        $V_2 = 1500$

	$\theta_3$	<u>A/B=1</u>	<u>A/B=1/2</u>	<u>A/B=1/5</u>	<u>A/B=1/10</u>	<u>A/B=2/1</u>	<u>A/B=5/1</u>
$\theta_1=10^\circ$	1. -40°	1.22%	0%	0%	0%	1.18%	2.20%
	2. -30	1.27	.51	0	0	1.64	1.87
	3. -20	1.31	.83	0	0	1.55	1.70
	4. -10	1.40	1.10	.20	0	1.54	1.63
	5. - 5	1.46	1.24	.58	0	1.57	1.63
	6. 0	1.54	1.39	.95	.20	1.61	1.65
	7. 5	1.64	1.56	1.34	.96	1.67	1.70
$\theta_1=20^\circ$	8. -10	1.31	.83	0	0	1.55	1.70
	9. - 5	1.40	.99	0	0	1.60	1.72
	10. 0	1.50	1.17	.17	0	1.66	1.76
	11. 5	1.61	1.36	.58	0	1.74	1.82
	12. 10	1.76	1.58	1.04	.12	1.85	1.90
	13. 15	1.93	1.84	1.55	1.06	1.98	2.01
$\theta_1=30^\circ$	14. - 5	1.37	.69	0	0	1.71	1.91
	15. 0	1.49	.89	0	0	1.79	1.97
	16. 5	1.63	1.11	0	0	1.89	2.05
	17. 10	1.79	1.35	0	0	2.02	2.15
	18. 15	1.99	1.63	.54	0	2.17	2.27
	19. 20	2.22	1.96	1.17	0	2.35	2.43
	20. 25	2.50	2.36	1.92	1.19	2.57	2.61
$\theta_1=40^\circ$	21. 0	1.49	.42	0	0	2.02	2.33
	22. 10	1.84	.93	0	0	2.28	2.55
	23. 20	2.30	1.60	0	0	2.65	2.86
	24. 30	2.99	2.55	1.21	0	3.21	3.34
	25. 35	3.46	3.21	2.45	1.16	3.59	3.66
$\theta_1=50^\circ$	26. 0	1.37	0	0	0	2.36	2.94
	27. 10	1.77	0	0	0	2.66	3.18
	28. 20	2.29	.68	0	0	3.08	3.55
	29. 30	3.05	1.72	0	0	3.70	4.09
	30. 40	4.23	3.35	.63	0	4.66	4.92
	31. 45	5.10	4.58	3.00	.24	5.35	5.50
$\theta_1=55^\circ$	32. 0	1.15	0	0	0	2.55	3.38
	33. 10	1.58	0	0	0	2.88	3.64
	34. 20	2.14	0	0	0	3.33	4.02
	35. 30	2.94	.78	0	0	3.99	4.61
	36. 40	4.18	2.48	0	0	5.01	5.51
	37. 50	6.34	5.53	3.04	0	6.73	6.97

$$V_1 = 1525$$

$$V_2 = 1500$$

	$\theta_3$	$A/B=1$	$A/B=1/2$	$A/B=1/5$	$A/B=1/10$	$A/B=2/1$	$A/B=5/1$
$\theta_1=60^\circ$	38. 0°	.66%	0%	0%	0%	2.73%	3.95%
	39. 10	1.14	0	0	0	3.08	4.21
	40. 20	1.73	0	0	0	3.56	4.63
	41. 30	2.58	0	0	0	4.27	5.27
	42. 40	3.89	.78	0	0	5.38	6.25
	43. 50	6.17	3.97	0	0	7.23	7.28
$\theta_1=65^\circ$	44. 0	0	0	0	0	2.79	4.67
	45. 10	.10	0	0	0	3.17	4.95
	46. 20	.75	0	0	0	3.70	5.40
	47. 30	1.63	0	0	0	4.47	6.10
	48. 40	2.99	0	0	0	5.66	7.19
	49. 50	5.39	.51	0	0	7.67	8.98
	50. 55	10.27	7.81	0	0	11.46	12.16

TABLE 3  
RELATIVE ERROR  
 $V_1 = 1525$        $V_2 = 1475$

	$\theta_3$	$A/B=1$	$A/B=1/2$	$A/B=1/5$	$A/B=1/10$	$A/B=2/1$	$A/B=5/1$
$\theta_1=10^\circ$	1. $-40^\circ$	2.43%	0%	0%	0%	3.65%	4.36%
	2. $-30$	2.54	1.02	0	0	3.28	3.72
	3. $-20$	2.63	1.67	0	0	3.10	3.39
	4. $-10$	2.80	2.21	.41	0	3.09	3.26
	5. $- 5$	2.92	2.49	1.17	0	3.13	3.26
	6. $0$	3.08	2.79	1.91	.41	3.22	3.31
	7. $5$	3.27	3.13	2.68	1.93	3.35	3.39
$\theta_1=20^\circ$	8. $-10$	2.63	1.67	0	0	3.10	3.39
	9. $- 5$	2.80	2.00	0	0	3.19	3.43
	10. $0$	2.99	2.34	.34	0	3.31	3.51
	11. $5$	3.23	2.72	1.18	0	3.48	3.63
	12. $10$	3.51	3.16	2.08	.25	3.68	3.79
	13. $15$	3.85	3.66	3.09	2.14	3.94	4.00
$\theta_1=30^\circ$	14. $- 5$	2.75	1.40	0	0	3.41	3.80
	15. $0$	2.98	1.79	0	0	3.57	3.92
	16. $5$	3.26	2.22	0	0	3.77	4.07
	17. $10$	3.58	2.70	0	0	4.01	4.26
	18. $15$	3.95	3.25	1.09	0	4.30	4.51
	19. $20$	4.40	3.90	2.35	0	4.65	4.80
	20. $25$	4.95	4.67	3.83	2.39	5.09	5.17
$\theta_1=40^\circ$	21. $0$	2.97	.84	0	0	4.01	4.62
	22. $10$	3.65	1.87	0	0	4.52	5.03
	23. $20$	4.56	3.18	0	0	5.24	5.64
	24. $30$	5.88	5.04	2.42	0	6.30	6.55
	25. $35$	6.78	6.31	4.85	2.33	7.02	7.16
$\theta_1=50^\circ$	26. $0$	2.72	0	0	0	4.64	5.77
	27. $10$	3.50	0	0	0	5.22	6.23
	28. $20$	4.53	1.37	0	0	6.04	6.92
	29. $30$	5.98	3.41	0	0	7.22	7.94
	30. $40$	8.21	6.56	1.26	0	9.02	9.49
	31. $45$	9.82	8.87	5.89	.49	10.29	10.57
$\theta_1=55^\circ$	32. $0$	2.28	0	0	0	5.01	6.58
	33. $10$	3.13	0	0	0	5.63	7.07
	34. $20$	4.23	0	0	0	6.49	7.81
	35. $30$	5.77	1.56	0	0	7.75	8.90
	36. $40$	8.12	4.89	0	0	9.66	10.56
	37. $50$	12.06	10.61	5.96	0	12.77	13.19

$$V_1 = 1525$$

$$V_2 = 1475$$

	<u><math>\theta_3</math></u>	<u>A/B=1</u>	<u>A/B=1/2</u>	<u>A/B=1/5</u>	<u>A/B=1/10</u>	<u>A/B=2/1</u>	<u>A/B=5/1</u>
$\theta_1=60^\circ$	38. 00	1.30%	0%	0%	0%	5.32%	7.62%
	39. 10	2.35	0	0	0	5.99	8.13
	40. 20	3.43	0	0	0	6.92	8.91
	41. 30	5.06	0	0	0	8.26	10.08
	42. 40	7.55	1.55	0	0	10.30	11.88
	43. 50	11.75	7.72	0	0	13.63	14.73
$\theta_1=65^\circ$	44. 0	0	0	0	0	5.40	8.90
	45. 10	.20	0	0	0	6.13	9.42
	46. 20	1.48	0	0	0	7.13	10.26
	47. 30	3.21	0	0	0	8.57	11.54
	48. 40	5.83	0	0	0	10.77	13.50
	49. 50	10.31	1.02	0	0	14.36	16.62
	50. 55	18.84	14.64	0	0	20.79	21.91

TABLE 4  
RELATIVE ERROR  
 $V_1 = 1500$        $V_2 = 1550$

	$\theta_3$	$A/B=1$	$A/B=1/2$	$A/B=1/5$	$A/B=1/10$	$A/B=2/1$	$A/B=5/1$
$\theta_1=10^\circ$	1. -40°	2.48%	0%	0%	0%	3.81%	4.62%
	2. -30	2.57	1.00	0	0	3.37	3.85
	3. -20	2.65	1.65	0	0	3.16	3.46
	4. -10	2.82	2.20	.40	0	3.13	3.32
	5. - 5	2.95	2.49	1.14	0	3.18	3.32
	6. 0	3.12	2.81	1.89	.39	3.27	3.37
	7. 5	3.33	3.17	2.70	1.91	3.41	3.46
$\theta_1=20^\circ$	8. -10	2.65	1.65	0	0	3.16	3.46
	9. - 5	2.82	1.98	0	0	3.25	3.51
	10. 0	3.03	2.34	.33	0	3.38	3.59
	11. 5	3.29	2.75	1.15	0	3.56	3.73
	12. 10	3.60	3.21	2.08	.24	3.79	3.91
	13. 15	3.98	3.77	3.15	2.13	4.08	4.14
$\theta_1=30^\circ$	14. - 5	2.79	1.38	0	0	3.50	3.94
	15. 0	3.04	1.79	0	0	3.68	4.07
	16. 5	3.34	2.23	0	0	3.90	4.24
	17. 10	3.69	2.74	0	0	4.17	4.46
	18. 15	4.11	3.33	1.07	0	4.50	4.74
	19. 20	4.62	4.05	2.36	0	4.91	5.08
	20. 25	5.26	4.93	3.97	2.41	5.42	5.52
$\theta_1=40^\circ$	21. 0	3.06	.83	0	0	4.21	4.91
	22. 10	3.80	1.89	0	0	4.79	5.38
	23. 20	4.83	3.28	0	0	5.63	6.11
	24. 30	6.40	5.39	2.46	0	6.91	7.22
	25. 35	7.52	6.92	5.17	2.36	7.82	8.00
$\theta_1=50^\circ$	26. 0	2.85	0	0	0	5.03	6.38
	27. 10	3.70	0	0	0	5.71	6.94
	28. 20	4.86	1.38	0	0	6.68	7.79
	29. 30	6.58	3.57	0	0	8.15	9.11
	30. 40	9.45	7.29	1.26	0	10.56	11.24
	31. 45	11.71	10.34	6.44	.48	12.40	12.82
$\theta_1=55^\circ$	32. 0	2.41	0	0	0	5.56	7.52
	33. 10	3.34	0	0	0	6.29	8.12
	34. 20	4.57	0	0	0	7.34	9.06
	35. 30	6.38	1.59	0	0	8.94	10.53
	36. 40	9.39	5.28	0	0	11.56	12.91
	37. 50	15.22	12.93	6.55	0	16.40	17.12



$$V_1 = 1500$$

$$V_2 = 1550$$

	$\theta_3$	$A/B=1$	$A/B=1/2$	$A/B=1/5$	$A/B=1/10$	$A/B=2/1$	$A/B=5/1$
$\theta_1=60^\circ$	38. 0°	1.39%	0%	0%	0%	6.10%	9.08%
	39. 10	2.42	0	0	0	6.90	9.74
	40. 20	3.74	0	0	0	8.06	10.80
	41. 30	5.63	0	0	0	9.82	12.47
	42. 40	8.76	1.59	0	0	12.70	15.20
	43. 50	14.84	8.88	0	0	18.06	20.08
$\theta_1=65^\circ$	44. 0	0	0	0	0	10.39	11.34
	45. 10	.22	0	0	0	11.83	12.08
	46. 20	1.63	0	0	0	13.90	13.31
	47. 30	3.57	0	0	0	17.11	15.29
	48. 40	6.72	0	0	0	22.51	18.55
	49. 50	12.88	1.04	0	0	33.02	24.43
	50. 55	28.92	19.98	0	0	60.45	37.17



TABLE 5  
RELATIVE ERROR  
 $V_1 = 1550$        $V_2 = 1500$

	$\theta_3$	$A/B=1$	$A/B=1/2$	$A/B=1/5$	$A/B=1/10$	$A/B=2/1$	$A/B=5/1$
$\theta_1=10^\circ$	1. -40°	2.39%	0%	0%	0%	3.59%	4.29%
	2. -30	2.50	1.00	0	0	3.23	3.66
	3. -20	2.59	1.65	0	0	3.05	3.33
	4. -10	2.75	2.18	.41	0	3.04	3.21
	5. - 5	2.87	2.45	1.15	0	3.08	3.21
	6. 0	3.03	2.74	1.88	.40	3.17	3.25
	7. 5	3.22	3.07	2.64	1.90	3.29	3.34
$\theta_1=20^\circ$	8. -10	2.59	1.65	0	0	3.05	3.33
	9. - 5	2.75	1.96	0	0	3.14	3.37
	10. 0	2.94	2.30	.33	0	3.26	3.45
	11. 5	3.17	2.68	1.16	0	3.42	3.57
	12. 10	3.45	3.10	2.05	.24	3.62	3.73
	13. 15	3.79	3.60	3.04	2.10	3.88	3.93
$\theta_1=30^\circ$	14. - 5	2.70	1.37	0	0	3.35	3.74
	15. 0	2.94	1.76	0	0	3.51	3.85
	16. 5	3.20	2.19	0	0	3.71	4.00
	17. 10	3.52	2.66	0	0	3.94	4.20
	18. 15	3.89	3.20	1.07	0	4.23	4.43
	19. 20	4.33	3.83	2.31	0	4.58	4.73
	20. 25	4.87	4.60	3.77	2.35	5.01	5.09
$\theta_1=40^\circ$	21. 0	2.93	.83	0	0	3.94	4.55
	22. 10	3.59	1.84	0	0	4.44	4.95
	23. 20	4.49	3.13	0	0	5.16	5.55
	24. 30	5.79	4.96	2.38	0	6.20	6.45
	25. 35	6.68	6.21	4.78	2.29	6.91	7.05
$\theta_1=50^\circ$	26. 0	2.68	0	0	0	4.57	5.68
	27. 10	3.45	0	0	0	5.14	6.14
	28. 20	4.46	1.35	0	0	5.94	6.82
	29. 30	5.89	3.36	0	0	7.11	7.82
	30. 40	8.09	6.46	1.24	0	8.88	9.35
	31. 45	9.67	8.73	5.80	.48	10.14	10.41
$\theta_1=55^\circ$	32. 0	2.24	0	0	0	4.93	6.48
	33. 10	3.08	0	0	0	5.54	6.96
	34. 20	4.16	0	0	0	6.39	7.69
	35. 30	5.68	1.53	0	0	7.63	8.77
	36. 40	8.00	4.81	0	0	9.51	10.40
	37. 50	11.89	10.45	5.87	0	12.59	13.00

$$V_1 = 1550$$

$$V_2 = 1500$$

	<u><math>\theta_3</math></u>	<u>A/B=1</u>	<u>A/B=1/2</u>	<u>A/B=1/5</u>	<u>A/B=1/10</u>	<u>A/B=2/1</u>	<u>A/B=5/1</u>
$\theta_1=60^\circ$	38. 0°	1.28%	0%	0%	0%	5.24%	7.50%
	39. 10	2.21	0	0	0	5.90	8.00
	40. 20	3.37	0	0	0	6.81	8.77
	41. 30	4.98	0	0	0	8.13	9.33
	42. 40	7.44	1.52	0	0	10.15	11.71
	43. 50	11.57	7.60	0	0	13.44	14.52
$\theta_1=65^\circ$	44. 0	0	0	0	0	5.32	8.77
	45. 10	.20	0	0	0	6.03	9.29
	46. 20	1.46	0	0	0	7.02	10.11
	47. 30	3.16	0	0	0	8.44	11.37
	48. 40	5.74	0	0	0	10.61	13.31
	49. 50	10.16	1.01	0	0	14.16	16.39
	50. 55	18.58	14.53	0	0	20.51	21.63

TABLE 6  
ABSOLUTE ERROR

$V_1 = 1500 \quad V_2 = 1525$					$V_1 = 1525 \quad V_2 = 1500$		
	$\theta_3$	$A/B=1$	$A/B=5/1$	$A/B=1/5$	$A/B=1$	$A/B=5/1$	$A/B=1/5$
$\theta_1=10^\circ$	1. $-40^\circ$	.03%	.23%	0%	.03%	.23%	0%
	2. $-30$	.03	.26	0	.04	.26	0
	3. $-20$	.05	.32	0	.05	.32	0
	4. $-10$	.08	.53	0	.08	.47	.03
	5. $-5$	.11	.62	.02	.11	.62	.09
	6. $0$	.17	.94	.08	.18	.96	.21
	7. $5$	.37	-	.22	.32	-	.61
$\theta_1=30^\circ$	8. $-5$	.04	.30	0	.04	.30	0
	9. $0$	.05	.36	0	.06	.36	0
	10. $5$	.07	.44	0	.07	.45	0
	11. $10$	.10	.58	0	.09	.58	0
	12. $15$	.14	.80	.08	.15	.81	.07
	13. $20$	.23	1.28	.24	.24	1.28	.24
	14. $25$	.52	2.29	.79	.51	2.73	.80
$\theta_1=50^\circ$	15. $0$	.03	.30	0	.03	.30	0
	16. $10$	.04	.39	0	.04	.39	0
	17. $20$	.07	.56	0	.07	.55	0
	18. $30$	.13	.93	0	.12	.92	0
	19. $40$	.35	2.07	.10	.32	2.09	.10
	20. $45$	.82	4.46	.95	.83	4.50	.96

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# Appendix I

```

PROGRAM THESIS3
C   ASSUME TWO BUOYS A DISTANCE OF 2A APART. ORIGIN (0,0) IS INBETWEEN
C   ACTUAL TARGET IS AT(X3,Y3), APPARENT TARGET AT(X4,Y4). ASSUME 2
C   DIFFERENT VELOCITY GRADIENTS V1 AND V2 WHERE V2 IS A DISTANCE B FROM
C   THE BUOY AXIS.
    READ 1000,K,V1,V2
1000 FORMAT(I5,F10.3,F10.3)
C   HOLD EITHER THETA 1 OR THETA 3 CONSTANT. K IS NUMBER OF VARYING
C   THETA OR A OR B TO BE READ IN.
    J=0
    L=K+1
  7 J=J+1
    IF(J.EQ.L)1,2
  2 READ 2000,A,B,THETA1,THETA3
2000 FORMAT(F6.3,F6.3,F10.6,F10.6)
    PRINT 5000,V1,V2
5000 FORMAT(F10.3,F10.3)
    PRINT 200,A,B,THETA1,THETA3
  200 FORMAT(1H119X,5H A = F6.3,10X,5H B = F6.3,10X,10H THETA1 = F10.6,
    *10X,10H THETA3 = F10.6//)
    IF(V1 .LT. V2)3,4
  3 THETAC=ASINF(V1/V2)
    PRINT 300,THETAC
  300 FORMAT(20X,10H THETAC = F10.6//)
    IF(THETA1 .LT. THETAC)5,6
  5 CONTINUE
  4 TT1=TANF(THETA1)
    TT3=TANF(THETA3)
    X4=2*A/(TT1-TT3)
    Y4=A*(TT1+TT3)/(TT1-TT3)
    PRINT 400,TT1,TT3
  400 FORMAT(20X,7H TT1 = F9.6//20X,7H TT3 = F9.6//)
    PRINT 500,X4,Y4
  500 FORMAT(20X,22H APPARENT POSITION = (F5.2,2H ,F5.2,2H )//)
    Z1=ASINF(V2/V1*SINF(THETA1))
    Z3=ASINF(V2/V1*SINF(THETA3))
    T1=TANF(Z1)
    T3=TANF(Z3)
    PRINT 600,Z1,Z3,T1,T3
  600 FORMAT(20X,6H Z1 = F9.6,10X,6H Z3 = F9.6,10X,6H T1 = F9.6,10X,
    *6H T3 = F9.6//)
    X3= (B*(TT3-TT1+T1-T3)+2*A)/(T1-T3)
    Y3=(B*(T3*TT1-T1*TT3)-A*(T1+T3))/(T3-T1)
    IF (X3 .LE. B)8,9
  8 X3=X4
    Y3=Y4
  9 PRINT 700,X3,Y3
  700 FORMAT(20X,18H REAL POSITION = (F5.2,2H ,F5.2,2H )//)
    E=(SQRTF((X3-X4)**2+(Y3-Y4)**2))*100.0/SQRTF(X3**2+Y3**2)
    PRINT 800,E
  800 FORMAT(20X,16H ERROR VECTOR = F5.2, 8H PERCENT)
    GO TO 7
  6 PRINT 900
  900 FORMAT(20X,53H ANGLE GREATER THAN CRITICAL ANGLE, TOTAL REFLECTION

```

\*.)  
GO TO 7  
1 CONTINUE  
END  
END  
FINIS  
-EXECUTE.



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<p>An investigation was made into the types of fixing error that would result when two passive directional sensors in one velocity medium was obtaining its fix on a target in another velocity medium. For typical velocity differences that could exist in the ocean off the U. S., this error was investigated for various values of sensor spacing and target/sensor relationships to a velocity discontinuity. Tabular and graphical results were obtained which revealed that errors could result under certain conditions and these results were predictable and could operationally be avoided.</p>			

14. KEY WORDS	LINK A		LINK B		LINK C	
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SONOBUOY ACOUSTICAL SENSORS HORIZONTAL GRADIENT FIX ERROR						

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